

UNIVERSITY OF CRAIOVA
Faculty of Automatics, Computers and Electronics
”Constantin Belea” Doctoral School
Field: System Engineering

PHD THESIS

(Abstract)

**Control Algorithms for Balancing Pendulum Models
with Elastic Components**

PhD Supervisor,

Prof. univ. dr. ing. Mircea IVĂNESCU

PhD Student,

Van Dong Hai NGUYEN

Craiova

2018

In this dissertation, author examines control algorithms for robots with elastic components. Two objects are concern is two-legged robot with elastic legs and elastic inverted pendulum. Their dynamic equations are generated and controllers are created to applied.

Firstly, mathematical model of kinds of inverted pendulum are presented. Based on these dynamic equations and some real experimental model in laboratories, simulation and experimental results under different kinds of controllers through kinds of inverted pendulum are generated. Conventional, nonlinear and intelligent controllers are tested in both simulation and experiment. Mainly, linear feedback are used as PD control and LQR control. Nonlinear control are also concerned. In this case, hierarchical sliding mode control is examined due to its successful operation on under-actuated SIMO system.

Then, from a similar form with IP, acrobot is concerned. Thence, based on acrobot structure, mathematical dynamic equations of a kind of two-legged robot with elastic legs are generated and analized. This robot can be considered to be closed to athlete robot. Due to the complexity of complete mathematical dynamic equation, an approximated equivalent model of robot is presented. Under this equivalent model, LQR and hierarchical sliding mode control are examined sucessfully on simulation, only. Robot can stand on one leg. By the uncertainty of equivalent model, hierarchical sliding mode control is proven to be more efficient in this case. Beside robot with elastic legs, mathematical model of elastic inverted pendulum are presented and analyzed. PD and hierarchical sliding mode controllers are applied for these robot. Genetic algorithm are used to find or optimize controllers in both simulation and experiment.

Also, real experimental platforms of elastic inverted pendulum and robot with elastic legs are presented and experimental results under linear feedback controllers are introduced. Then, conclusion which summerized the content of thesis, the direction in the future for athlete robot object and methods ends the thesis.

Contents

Contents	iii
Chapter 1 : INVERSE PENDULUM – BASIC MODEL OF ROBOT SYSTEM	1
Chapter 2: INVERSE PENDULUM-DYNAMICS	3
Chapter 3: LYAPUNOV BASED ALGORITMS FOR INVERSE PENDULUM MODELS	6
3.1. Lyapunov method for Cart and Pole.....	6
3.2. Robust control.....	6
3.3. Fuzzy-Lyapunov based Algorithm for IP models	7
Chapter 4: FUZZY CONTROL FOR INVERSE PENDULUM MODELS	8
4.1. Lyapunov Method based Fuzzy Controller.	8
4.2. Hybrid Controller.	9
Chapter 5: INVERSE PENDULUM MODELS WITH ELASTIC COMPONENTS	11
5.1. Elastic Inverted pendulum.....	11
5.2. Elastic C-shaped Leg Models.....	13
5.3. C-shaped Leg Robot Control by Lyapunov Methods.	14
Chapter 6: JUMPING MOTION CONTROL ALGORITHMS	17
6.1. Stance Phase Model	18
6.2. Stance Phase: Touch-Down Sequence	19
6.3. Stance Phase: Take-off Sequence	25
Chapter 7: SIMULATION OF CONTROL ALGORITHMS	29
7.1. LQR Control Simulation of E-IP Model.	29
7.2. HSM Control for E-IP System.....	29
7.3. Conventional PD Control for Two-Legged Robot.....	30
Chapter 8: EXPERIMENTAL STUDY OF MODELS WITH ELASTIC COMPONENTS	32
8.1. Elastic Inverted Pendulum	32
8.2. Two-legged robot with Elastic legs.....	33
REFERENCE	36
LIST OF PUBLICATIONS	44

Chapter 1 : INVERSE PENDULUM – BASIC MODEL OF ROBOT SYSTEM

IP is a basic model for control theory. Its history was begun by construction of Robege (1960). Then, some representative researchers, such as, Schaefer and Cannon (1966), Furuta et al. (1991) developed basic theory for this model [1]. In this period, researches on pendulum mostly are mathematical problems and this model is not popular as a model for widely researching in control engineering. But, after years of survey on theory and mathematics on IP, Furuta invented Rotary IP – Furuta pendulum – and opened the ability of real experiment for this kind of model [2]. Since then, a lot of researches are developed based on this pioneer invention. Its simple mechanical structure – with only one motor and two simple sensors which in most cases are two encoders [3] – makes fabrication to be possible in usual laboratories, even in undeveloped countries. Moreover, nonlinear and under-actuated characteristics in mathematics makes it an ideal model for most control theory experiment. And very fast, following this achievement, big amount of algorithms were tested both in algorithm theory and experiment [4], [5].

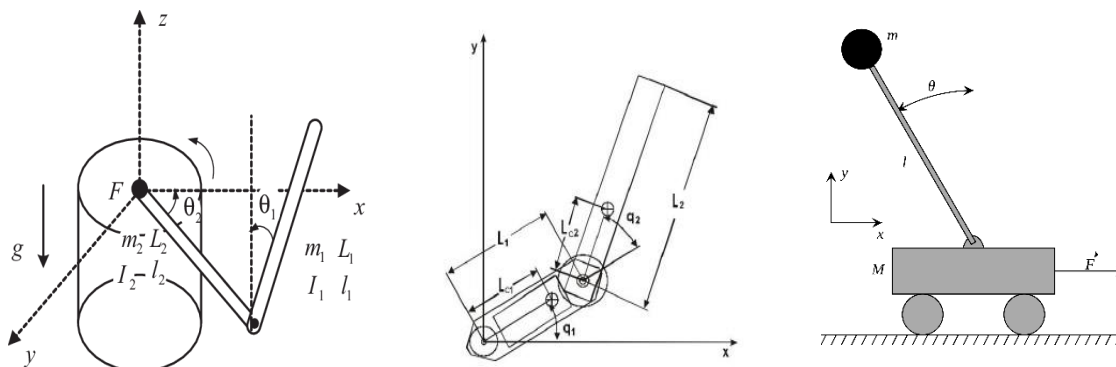


Fig 1.3 : Products stimulated by IP controlling

A lot of control algorithms are utilized to balance IP [4]. PID control [64], [65], has been proven to have good performances and control parameters are based on trial and error method [65] or searching algorithm, such as genetic algorithm [66] or LQR [36], [14], pole-placement control [67], [68]. The structure of these algorithms is simple and offers a lot of facilities for embedded systems. Fuzzy controller, which was presented by Zadeh in 1965 [69], represents also a good solution for developing new implementation techniques. Nonlinear control, especially SMC [70], introduces new robust algorithms that ensure a good stability of motion [71]. Hybrid controllers [18], [19], [59] have been presented to combine the advantages of intelligent systems with non linear algorithms.

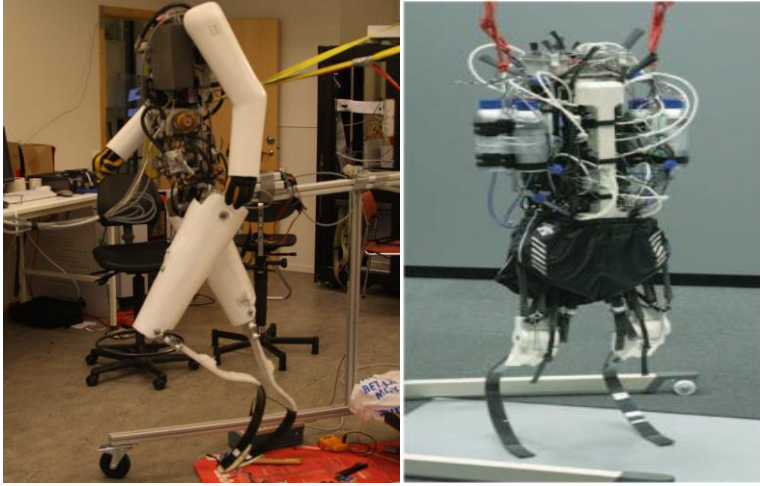


Figure 1.1: Dasher robot (left) and AR in Tokyo University (right)

Chapter 2: INVERSE PENDULUM-DYNAMICS

Beside the classical model-IP on cart, some other developed models are presented: more links are added to create double IP; changing in mechanical structure creates pendubot; replacing the position of actuator in pendubot creates acrobot. Dynamic equations are generated and they are the base for control algorithms in following sections. Under dynamic equations, a simple linear feedback controller is applied in some models before any other survey on other kinds of controllers. IP on cart (or Cart and Pole system) concludes of a cart, which moves in horizontal direction, and a pole, which rotates around an axis on cart.

a) Case 1: Distributed Mass Pendulum

We consider that the mass of pendulum is distributed along the length (Figure 2.1).

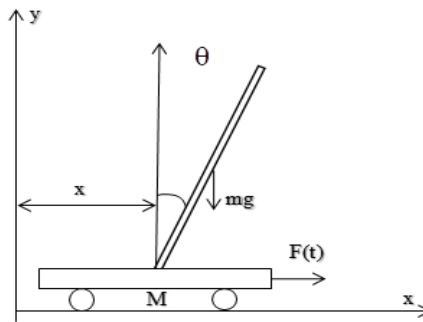


Figure 2.1: Cart and pole system with the pendulum as homogeneity

The dynamic model is inferred as

$$\ddot{x} = \frac{1}{\Gamma_1 + \Gamma_2 \sin^2 \theta} \left[\Gamma_2 \sin \theta (L\dot{\theta}^2 - g \cos \theta) + F \right] \quad (2.1)$$

$$\ddot{\theta} = \frac{1}{L(\Gamma_1 + \Gamma_2 \sin^2 \theta)} \left[-\Gamma_2 L\dot{\theta}^2 \sin \theta \cos \theta + (\Gamma_1 + \Gamma_2) g \sin \theta - F \cos \theta \right] \quad (2.2)$$

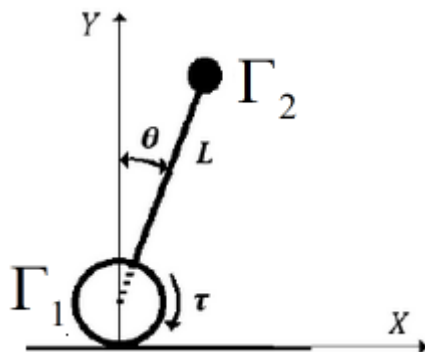


Figure 2.2: Balancing robot on wheel

For the balancing robot, equation of the system can be described as:

$$\ddot{x} = \frac{\Gamma_2 \sin \theta (L\dot{\theta}^2 - g \cos \theta) + \tau/r}{\Gamma_1 + \Gamma_2 \sin^2 \theta} \quad (2.3)$$

$$\ddot{\theta} = \frac{-\Gamma_2 L \dot{\theta}^2 \sin \theta \cos \theta + (\Gamma_1 + \Gamma_2) g \sin \theta - \tau \cos \theta / r}{L(\Gamma_1 + \Gamma_2 \sin^2 \theta)} \quad (2.4)$$

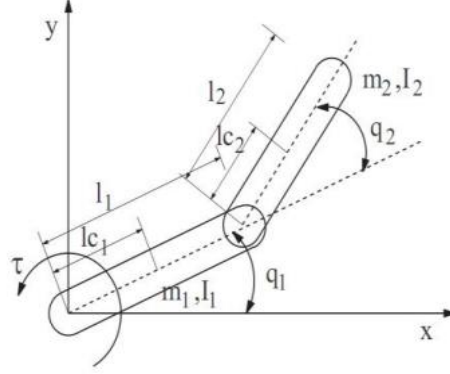


Figure 2.3: Mathematical structure of Pendubot

The dynamic equations of Pendubot are inferred as

$$\ddot{q}_1 = \frac{\left[\begin{array}{l} \beta_2 \tau_1 + \beta_2 \beta_3 (x_2 + x_4)^2 \sin x_3 + \beta_3^2 x_2^2 \sin x_3 \cos x_3 + \\ -\beta_2 \beta_4 g \cos x_1 + \beta_3 \beta_5 g \cos x_3 \cos (x_1 + x_3) \end{array} \right]}{\beta_1 \beta_2 - \beta_3^2 \cos^2 x_3} \quad (2.5)$$

$$\ddot{q}_2 = \frac{\left[\begin{array}{l} (-\beta_2 - \beta_3 \cos x_3) \tau_1 + \beta_4 g (\beta_2 + \beta_3 \cos x_3) \cos x_1 + \\ -\beta_3 (\beta_2 + \beta_3 \cos x_3) (x_2 + x_4)^2 \sin x_3 + \\ -\beta_5 g (\beta_1 + \beta_3 \cos x_3) \cos (x_1 + x_3) - \beta_3 x_2^2 \sin x_3 (\beta_1 + \beta_3 \cos x_3) \end{array} \right]}{\beta_1 \beta_2 - \beta_3^2 \cos^2 x_3} \quad (2.6)$$

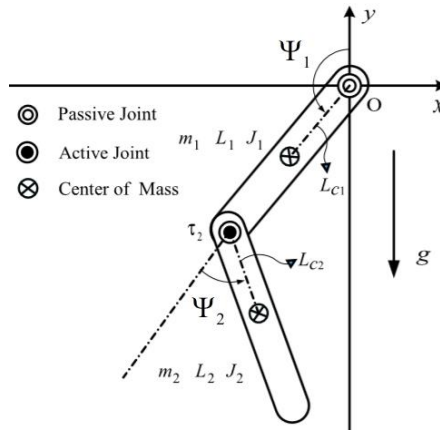


Figure 2.4: Model of acrobot

Considering that the friction is very small, the dynamic equations are

$$\frac{d}{dt} \left[\frac{\partial L(\Psi, \dot{\Psi})}{\partial \dot{\Psi}_i} \right] - \frac{\partial L(\Psi, \dot{\Psi})}{\partial \Psi_i} = \tau_i, (i = 1, 2) \quad (2.7)$$

Because $\tau_1 = 0$ due to mechanical structure of acrobot, (2.7) becomes

$$M(\Psi_2)\ddot{\Psi} + C(\Psi, \dot{\Psi})\dot{\Psi} + G(\Psi) = [0 \quad \tau_2]^T \quad (2.8)$$

where

$$C(\Psi, \dot{\Psi}) = \begin{bmatrix} -\Phi_3 \dot{\Psi}_2 \sin \Psi_2 & -\Phi_3 (\dot{\Psi}_1 + \dot{\Psi}_2) \sin \Psi_2 \\ \Phi_3 \dot{\Psi}_1 \sin \Psi_2 & 0 \end{bmatrix};$$

$$G(\Psi) = \begin{bmatrix} G_1(\Psi) \\ G_2(\Psi) \end{bmatrix} = \begin{bmatrix} -\Phi_4 \sin \Psi_1 - \Phi_5 \sin(\Psi_1 + \Psi_2) \\ -\Phi_5 \sin(\Psi_1 + \Psi_2) \end{bmatrix}$$

Chapter 3: LYAPUNOV BASED ALGORITHMS FOR INVERSE PENDULUM MODELS

3.1. Lyapunov method for Cart and Pole

Consider the dynamic model of the Cart and Pole system.

Theorem 3.1: For the system that describes this dynamic model, if the control law is

$$u = \frac{\sigma_1}{\varepsilon_1} x_1 + \frac{\sigma_2}{\varepsilon_2} x_2$$

where the coefficients $\sigma_1 > 0$, $\sigma_2 > 0$, α , β , γ satisfy the following conditions:

$$\sigma_1 > \varepsilon_{1\max} \quad (3.2)$$

$$\sigma_1 + \sigma_2 \delta > \alpha + \varepsilon_{1\max} \beta \quad (3.3)$$

$$(\varepsilon_{1\min} - \sigma_1) \delta + \frac{1}{4} (\sigma_1 + \sigma_2 \delta - \alpha - \varepsilon_{1\max} \beta) < 0 \quad (3.4)$$

$$\delta - \sigma_1 \beta + \varepsilon_2 \beta + \varepsilon_2 \beta \eta_1 \eta_2 + \sigma_1 + \sigma_2 \delta - \alpha - \varepsilon_{1\max} \beta < 0 \quad (3.5)$$

$$\alpha > \frac{\delta}{4} > 0 \quad (3.6)$$

$$\beta > 2\delta \quad (3.7)$$

The system is asymptotically stable.

3.2. Robust control

Consider the dynamic equations of IP model and the dynamic equations are presented in matrix form:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \varepsilon_1 & -\varepsilon_2 x_1 x_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -\varepsilon_3 \end{bmatrix} u \quad (3.8)$$

where the state constraints (sector-type) are defined as

$$\begin{aligned} -\eta_1 &\leq x_1 \leq \eta_1; \\ -\eta_2 &\leq x_2 \leq \eta_2 \end{aligned} \quad (3.9)$$

Theorem 3.2: Consider the IP model (3.8) and control law

$$u = -ky \quad (3.10)$$

where the range of variables is constrained by (3.9)

If there exists parameters $\gamma, k, c_1, c_2, c_3, c_4, c_5, c_6$ that satisfy these conditions

$$0 < \gamma \leq 4 \quad (3.11)$$

$$0 < k < \frac{1}{2\sqrt{\gamma}} \quad (3.12)$$

$$0 \leq \gamma + 2\operatorname{Re}\left\{\left(\frac{C}{2}\right)^T (j\omega I - A)^{-1} B\right\} \quad (3.13)$$

then, this model is asymptotically stabilized

3.3. Fuzzy-Lyapunov based Algorithm for IP models

Consider a nonlinear system of IP model described by

$$\dot{x} = \Delta f(x) + f(x) + (b + \Delta b)u, \quad x(0) = x_0 \quad (3.14)$$

where: $\Delta f(x)$ and Δb represent the uncertainty of $f(x)$ and b , respectively

Fuzzy model can be described by r fuzzy rules. The i^{th} rule is

If z_1 is F_{i1} and z_2 is F_{i2} and ... and z_p is F_{ip} then

$$\dot{x} = (B^i + \Delta B^i)u + (A^i + \Delta A^i)x \quad (3.15)$$

$$F_{ij}(x_i) = \begin{cases} 1 - \frac{|x_i - j_i \tilde{\Delta}|}{\tilde{\Delta}} & |x_i - j_i \tilde{\Delta}| < \tilde{\Delta} \\ 0 & \text{elsewhere} \end{cases} \quad (3.16)$$

Theorem 3.3: Consider the control law is PD form with k is feedback matrix. If following conditions are verified:

a) $k_{\min} \leq k \leq k_{\max}$

b) $\operatorname{Re}\left\{c^T (j\omega I - H)^{-1} \hat{b}\right\} + (k_{\max}^{-1} - \varepsilon^{-1}\beta) \geq 0$

where $H = (A^1 + \nu I - k_{\min} M)$ is Hurwitz and $\hat{b} = b^1 - d/k$

c) The pair (H, \hat{b}) is controllable

Then, this model is asymptotically stabilized.

Chapter 4: FUZZY CONTROL FOR INVERSE PENDULUM MODELS

4.1. Lyapunov Method based Fuzzy Controller.

The knowledge of experts and the GA method do not guarantee the mathematical stability system under fuzzy controller. Thence, Lyapunov criterion can be used to design a controller. We consider the IP model, Lyapunov function is selected as:

$$V = \frac{1}{2}(\alpha x_1^2 + \beta x_2^2 + 2\delta x_1 x_2) \geq \frac{1}{2} \left[\left(\alpha - \frac{\delta}{2} \right) x_1^2 + (\beta - 2\delta) x_2^2 \right] \quad (4.1)$$

The derivative with respect time is

$$\dot{V} = \chi - \varepsilon_3 \mathcal{G} u \quad (4.2)$$

where $\chi = 5\varepsilon_1 x_1 x_2 + 2x_2^2 - 5\varepsilon_2 x_1 x_2^3 - 2\varepsilon_2 x_1^2 x_2^2 + 2x_1 x_2$; $\mathcal{G} = 2x_1 + 5x_2$

By using Lyapunov method, the stability of motion is obtained if the function (4.2) is negative definite. The new fuzzy controller has to implement these conditions.

Table 1: Selection condition of control signal to satisfy Lyapunov criterion

Condition of variables			Condition of control signal to keep system stable
$\mathcal{G} > 0$	$x_1 x_2 > 0$	$\chi \geq 0$	$u \geq \chi / (\varepsilon_{3\min} \mathcal{G})$
		$\chi < 0$	$u \geq \chi / (\varepsilon_{3\max} \mathcal{G})$
	$x_1 x_2 < 0$	$\chi \geq 0$	$u \geq \chi / (\varepsilon_{3\min} \mathcal{G})$
		$\chi < 0$	$u \geq \chi / (\varepsilon_{3\max} \mathcal{G})$
$\mathcal{G} < 0$	$x_1 x_2 > 0$	$\chi \geq 0$	$u \leq \chi / (\varepsilon_{3\min} \mathcal{G})$
		$\chi < 0$	$u \leq \chi / (\varepsilon_{3\max} \mathcal{G})$
	$x_1 x_2 < 0$	$\chi \geq 0$	$u \leq \chi / (\varepsilon_{3\min} \mathcal{G})$
		$\chi < 0$	$u \leq \chi / (\varepsilon_{3\max} \mathcal{G})$

Memberships of variables x_1 and x_2 are shown in Figure 4.1 and Figure 4.2. From column 1, 2 of Figure 4.3 and Table 1, the range of controller is shown in the column 3 of Table 1. Then, appropriate memberships for output are selected in Figure 4.3.

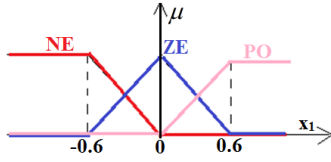


Figure 4.1: Memberships of x_1

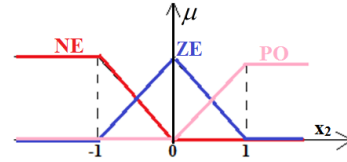


Figure 4.2: Memberships of x_2

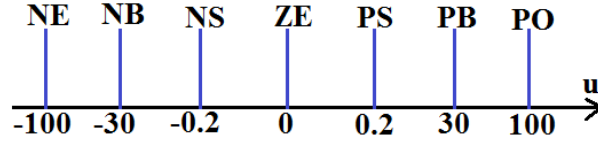


Figure 4.3: Memberships of output

4.2. Hybrid Controller.

Sliding Mode Control represents a very good technique for controlling the nonlinear systems by fuzzy techniques. Also, this method can be developed as a hierarchical structure for cascade connections of the IP models [54]-[58].

Assume that a dynamic equations of a Single Input Multiple Output (SIMO) system are

$$\ddot{\alpha}_i = A_i(\alpha_i) + B_i(\alpha_i)u \quad (4.3)$$

where α_i, α_{di} : state variables, reference signal of each variable; u : control input signal

$$\text{Denote: } e_i = \alpha_i - \alpha_{di} \quad (4.4)$$

as the error between state variable and reference signal

From (4.3), (4.4) the equivalent equations of system will be

$$\ddot{e}_i = f_i + g_i u \quad (4.5)$$

A controller can be designed for system (4.5) to stabilize variables $e_i \xrightarrow{t \rightarrow \infty} 0$

or $\alpha_i \xrightarrow{t \rightarrow \infty} \alpha_{id}$.

In Figure 4.4, a structure of hierarchical sliding surfaces is presented.

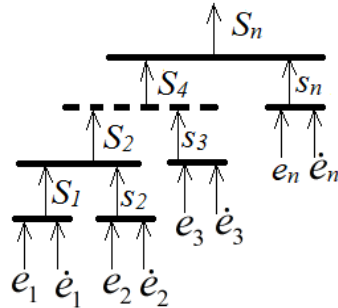


Figure 4.4: Hierarchical sliding surfaces structure for system

Sliding surfaces are denote as

$$s_k = c_k e_k + \dot{e}_k \quad (k \leq n) \quad (4.6)$$

$$S_k = a_{k-1}S_{k-1} + s_k \quad (k \leq n) \quad (4.7)$$

where $a_{i-1} = \text{const}$; $a_0 = S_0 = 0$.

$$S_k = \sum_{r=1}^k \left(\prod_{j=r}^k a_j \right) s_r \quad (k \leq n) \quad (4.8)$$

From (4.8), it yields

$$\dot{S}_k = \sum_{r=1}^k \left(\prod_{j=r}^k a_j \right) \dot{s}_r \quad (k \leq n) \quad (4.9)$$

The k^{th} layer SMC law is defined as

$$u_k = u_{k-1} + u_{eqk} + u_{svk} \quad (k \leq n) \quad (4.10)$$

u_{svk} , u_{eqk} is defined as switching and equivalent control law for k^{th} layer.

The final control signal is inferred

$$u_n = \frac{\left[\sum_{r=1}^n \left(\prod_{j=r}^n a_j \right) b_r u_{eqr} - \eta_n \text{sgn} S_n - \xi_n S_n \right]}{\sum_{r=1}^n \left(\prod_{j=r}^n a_j \right) b_r} \quad (4.11)$$

Theorem 4.1 For these dynamic equations, the sliding surfaces and control law specified in the previous part, the surface S_k is asymptotically stable.

Chapter 5: INVERSE PENDULUM MODELS WITH ELASTIC COMPONENTS

5.1. Elastic Inverted pendulum

In the previous sections, The IP models [105], [106], are based on the assumption that the pendulum bar is rigid. In all these cases, the model dynamics are defined by lumped parameter systems. In real applications, the mechanical architecture of the IP model contains flexible bars (Elastic IP model-E-IP model) that impose a new mathematical treatment, in which the bar dynamics are described by Partial Differential Equations (PDE).

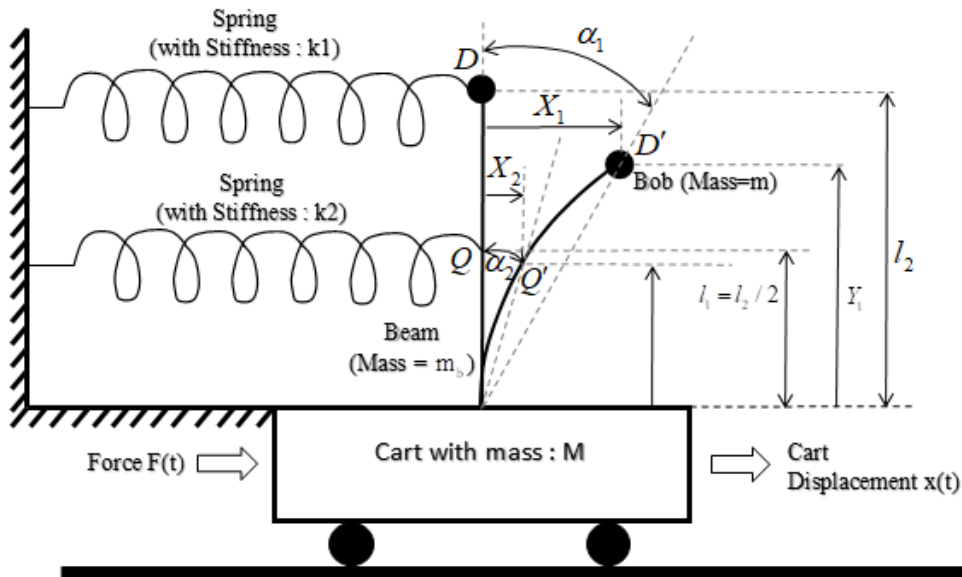


Figure 5.1: E-IP with tip mass is fixed on cart

Illustrative example of E-IP models are shown in Figure 5.1, Figure 5.2. In literature, few researchers investigate the control problem of these models [73]-[75] but they do not use a rigorous mathematical support for the controller design. Also, the performances achieved by proposed algorithms do not prove the quality of controllers.

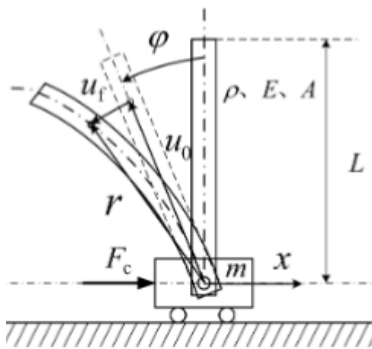


Figure 5.2: E-IP un-fixed on Cart

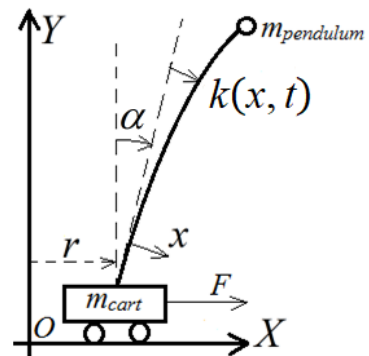


Figure 5.3: E-IP on Cart

According the Hamilton's principle

$$\int_{t_1}^{t_2} (\delta T - \delta V + \delta W_{nc}) dt = 0$$

where δT , δV , δW_{nc} represent the variational components of the kinetic, potential and non-conservative work.

The dynamic model will be

$$\begin{aligned} & -\left(m_{cart} + \rho l + m_{pendulum}\right) \ddot{r} - \left(m_{pendulum} l + \rho \frac{l^2}{2}\right) \ddot{\alpha} \cos \alpha + \\ & + m_{pendulum} \left[\ddot{\alpha} \times k(l,t) \sin \alpha - \dot{k}(l,t) \times \cos \alpha + \right. \\ & \left. + 2\dot{\alpha} \times \dot{k}(l,t) \sin \alpha + \dot{\alpha}^2 \times k(l,t) \times \cos \alpha \right] + \\ & + \left(\rho \frac{l^2}{2} + m_{pendulum} l\right) \dot{\alpha}^2 \sin \alpha + \rho \int_0^l \left(\ddot{\alpha} k \times \sin \alpha - \dot{k} \times \cos \alpha + \right. \\ & \left. + 2\dot{\alpha} \dot{k} \times \sin \alpha + \dot{\alpha}^2 k \times \cos \alpha \right) dx + F = 0 \end{aligned}$$

$$\begin{aligned} & -\left(J + m_{pendulum} l^2 + \rho \frac{l^3}{3}\right) \ddot{\alpha} - \left(m_{pendulum} l + \rho \frac{l^2}{2}\right) \ddot{r} \times \cos \alpha + \\ & + m_{pendulum} \left[\ddot{r} k(l,t) \times \sin \alpha - \ddot{\alpha} k^2 \times (l,t) - l \ddot{k}(l,t) + \right. \\ & \left. - 2\dot{\alpha} \times k(l,t) \times \dot{k}(l,t) + g k(l,t) \times \cos \alpha \right] + \\ & + \rho \int_0^l \left(\ddot{r} k \times \sin \alpha - \ddot{\alpha} k^2 - \ddot{k} x - 2\dot{\alpha} \dot{k} k + g k \times \cos \alpha \right) dx + \\ & + \left(m_{pendulum} g l + \rho g \frac{l^2}{2}\right) \sin \alpha = 0 \end{aligned}$$

$$m_{pendulum} \left[k(l,t) \times \dot{\alpha}^2 - \ddot{k}(l,t) - l \ddot{\alpha} - \ddot{r} \times \cos \alpha + g \sin \alpha \right] + EI \times k'''(l,t) = 0$$

$$\rho \left(k \dot{\alpha}^2 - \ddot{k} - \ddot{\alpha} x - \ddot{r} \times \cos \alpha + g \times \sin \alpha \right) - EI \times k''' = 0$$

$$k'''(0,t) = k''(0,t) = k''(l,t) = 0$$

Define $\Psi_i(t)$, $X_i(x)$ as function of time of i^{th} mode shape and function of mode shape of point x and $k(x,t)$ can be assumed to be $k(x,t) = \sum_{i=1}^n \Psi_i(t) X_i(x)$ [76]. We take into account only the first mode-shape. The dynamic equations of system are:

$$-\left(m_{cart} + \rho l + m_{pendulum}\right) \ddot{r} - \left(m_{pendulum} l \rho \frac{l^2}{2}\right) \ddot{\alpha} \cos \alpha + m_{pendulum} \left[\begin{array}{l} X(l) \Psi \ddot{\alpha} \sin \alpha + \\ -X(l) \dot{\Psi} \cos \alpha + \\ +2X(l) \dot{\Psi} \dot{\alpha} \sin \alpha + \\ +X(l) \Psi \dot{\alpha}^2 \cos \alpha \end{array} \right] + \quad (5.6)$$

$$+\left(\rho \frac{l^2}{2} + m_{pendulum} l\right) \dot{\alpha}^2 \sin \alpha + \rho \xi_1 \Psi \ddot{\alpha} \sin \alpha - \rho \xi_1 \ddot{\Psi} \cos \alpha + 2\rho \xi_1 \dot{\Psi} \dot{\alpha} \sin \alpha + \\ +\rho \xi_1 \Psi \dot{\alpha}^2 \cos \alpha + F = 0$$

$$-\left(J + m_{pendulum} l^2 + \rho \frac{l^3}{3}\right) \ddot{\alpha} - \left(m_{pendulum} l + \rho \frac{l^2}{2}\right) \ddot{r} \cos \alpha + \rho g \xi_1 \cos \alpha \Psi + \quad 7)$$

$$+\left(m_{pendulum} g l + \rho g \frac{l^2}{2}\right) \sin \alpha + m_{pendulum} \left[\begin{array}{l} X(l) \Psi \ddot{r} \sin \alpha - X^2(l) \Psi^2 \ddot{\alpha} + \\ -X(l) \ddot{\Psi} l - 2X^2(l) \Psi \dot{\Psi} \dot{\alpha} + \\ +X(l) \Psi g \cos \alpha \end{array} \right] +$$

$$+\rho \xi_1 \ddot{r} \Psi \sin \alpha - \rho \xi_2 \Psi^2 \ddot{\alpha} - \rho \xi_3 \ddot{\Psi} - 2\rho \xi_2 \Psi \dot{\Psi} \dot{\alpha} = 0$$

$$m_{pendulum} \left[X(l) \Psi \dot{\alpha}^2 - X(l) \ddot{\Psi} - l \ddot{\alpha} - \ddot{r} \cos \alpha + g \sin \alpha \right] + \xi_4 \Psi = 0 \quad 8)$$

5.2. Elastic C-shaped Leg Models.

C-shaped legs represent a special architecture of compliant elastic legs have been introduced to make the motion of two-legged robot more flexible



Figure 5.4: Curved beam

$$\text{Shear:} \quad F_r = F \cos \theta \quad (5.9)$$

$$\text{Axial:} \quad F_\theta = F \sin \theta \quad (5.10)$$

$$\text{Bending moment:} \quad M = FR \sin \theta \quad (5.11)$$

Strain energy due to bending moment M is

$$U_1 = \int \frac{M^2}{2AeE} d\theta \quad (5.12)$$

where e is eccentricity which is calculated as $e = R - r_n$. If assuming that $\frac{R}{h} > 10$,

it yields

$$U_1 = \int \frac{M^2 R}{2EI} d\theta \quad (5.13)$$

Strain energy due to axial force F_θ is

$$U_2 = \int \frac{F_\theta^2 R}{2AE} d\theta \quad (5.14)$$

Strain energy due to moment which is created by axial force F_θ is

$$U_3 = -\int \frac{MF_\theta}{AE} d\theta \quad (5.15)$$

The sign “-” in (5.15) is caused by the opposite direction of deflection to force F

$$U_4 = C \int \frac{F_r^2 R}{2AG} d\theta \quad (5.16)$$

The Hook’s law for linear spring is

$$F = k\delta \quad (5.17)$$

where: F is the force that applies on spring through the length of spring; k is the constant of the spring; δ is the deflection of the spring.

When consider C-shaped leg, the parameter k is not the constant when regarding this kind of leg as a linear spring. The value of k is different in each situation of leg when touching the ground.

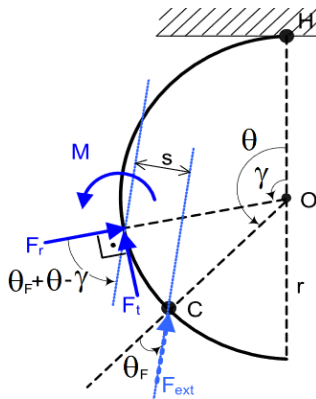


Figure 5.5: Cross section of C-shaped leg

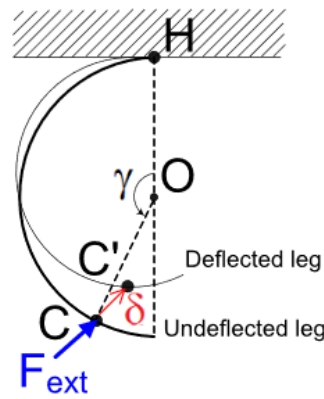


Figure 5.6: Effect of external force to C-shaped leg

5.3. C-shaped Leg Robot Control by Lyapunov Methods.

Considering that two legs of robot have the same behavior, motion of robot with elastic legs is divided into two phases: stance and flight phase. In stance phase, legs touch the ground in all period. Otherwise, legs are off ground in all period of flight phase. A PD controller which is designed based on Lyapunov stability’s theorem is surveyed.

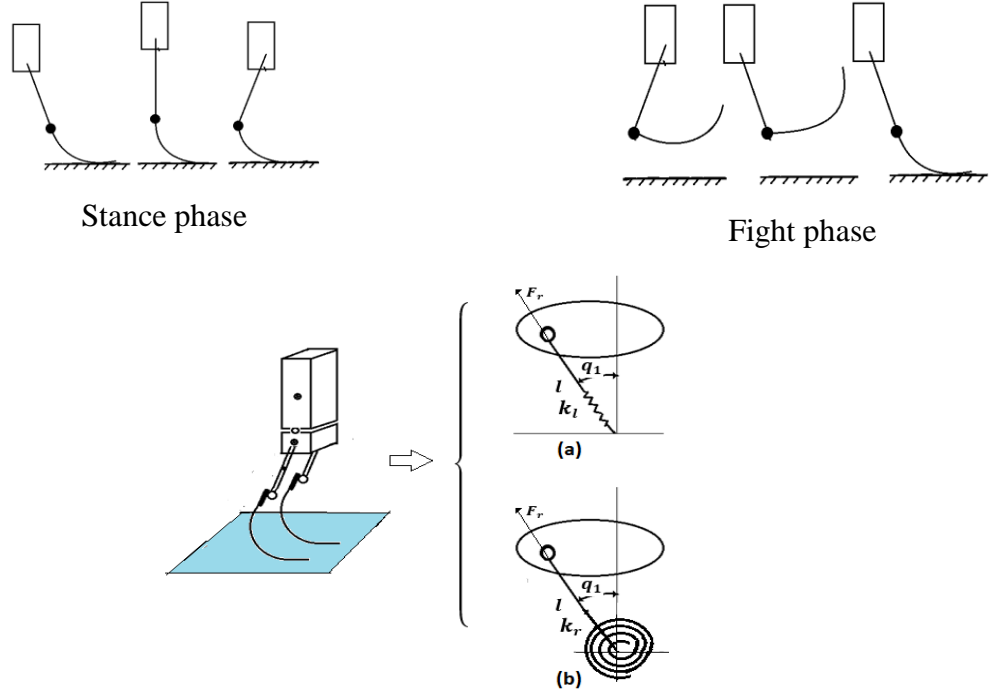


Figure 5.7: Equivalent IP model of robot with compliant legs, where:
a/ linear spring/b)rotational spring

Rotational deflection of elastic leg is

$$\delta_r \approx \frac{\partial U}{\partial M} \quad (5.18)$$

It yields

$$k_l \approx \frac{M}{\delta_l} = \frac{4EI}{R^2 \left\{ \pi + \frac{1}{2} [\sin(2\varphi) + \sin(2[\pi - \varphi])] \right\}}$$

Considering the elastic leg is hard and variation of shape in linear spring is small, by using Lagrange method, equations of stance phase in Figure 5.7b are obtained as

$$\left(M^* l^2 + I^* \right) \ddot{q}_1 = M^* g l \sin q_1 - k_r(\varphi) q_1 + \tau_1^* + h(\tau_2, \tau_3, q_2, q_3) \quad (5.19)$$

A PD linear feedback controller is suggested to be used in this case

$$\tau = -\mu_1 q_1 - \mu_2 \dot{q}_1 \quad (5.20)$$

Theorem 5.1: If system (5.19) is controlled by (5.20) where the control parameters satisfy the conditions

$$\sqrt{\frac{k_r}{N}} > \alpha > 0, \eta > 0 \quad (5.21)$$

$$\mu_1 > 0; \mu_2 > 0 \quad (5.22)$$

$$\begin{bmatrix} \mu_2 - \alpha \mathbb{N} - \eta & \frac{1}{2}(\mu_2 + \alpha \mu_2 - mgl) \\ \frac{1}{2}(\mu_1 + \alpha \mu_2 - mgl) & \alpha(k_{r\max} + \mu_1 - mgl - \eta) \end{bmatrix} > 0 \quad (5.23)$$

where: $\mathbb{N} = M^* l^2 + I^*$

then, this system is stable

Chapter 6: JUMPING MOTION CONTROL

ALGORITHMS

A multi-body robot system in Figure 6.1 was concerned to survey the jumping behaviour. There are two legs to determine the motion. Each leg concludes of two components:

- the lower part (foot) with a hybrid pneumatic/electro-hydraulic actuator and ER-fluid damper.
- The upper leg with a conventional electric drive.

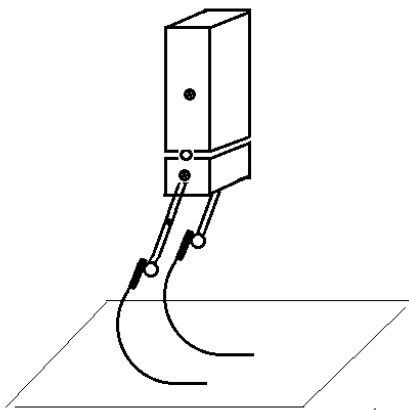


Figure 6.1: Model structure of jumping robot

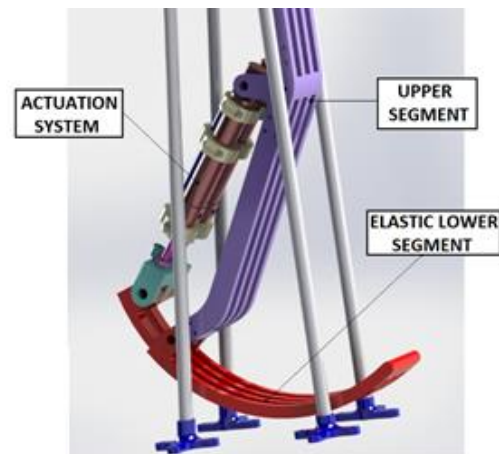


Figure 6.2: Platform of jumping robot

In order to simplify the technological problem, we considering that in jumping motion, two legs have the same posture. The mathematical model is not used and the structure of model in this section is described in Figure 6.2.

Similarly to walking model, jumping concludes two phases but these phases are defined differently: stance and flight phase. In stance phase, one foot touches the ground. And, in flight phase, both legs are off the ground. The boundary of these two phases are defined as two motions conditions: touch-down and take-off. These motions create a sequence when robot hits the ground (landing impact sequence) and when robot takes off the ground. This cycle is repeated periodically by robot (Figure 6.3, Figure 6.4). The feet is assumed not to slip beyond surface of the ground and always stay on contact with the ground when touching the ground.

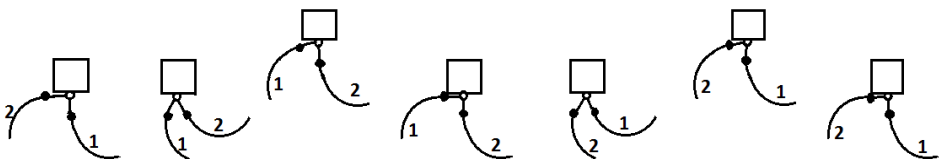


Figure 6.3: Cycle of motion

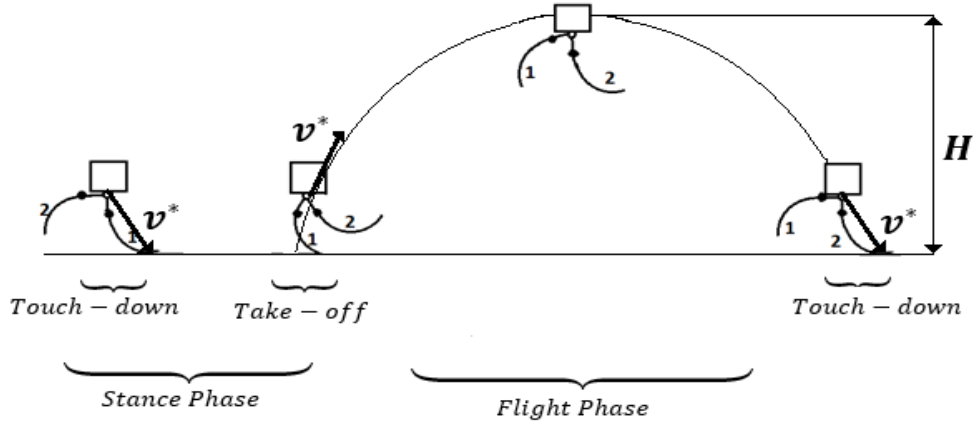


Figure 6.4: Trajectory of jumping motion

6.1. Stance Phase Model

This phase is determined by contact between the ground and the leg in Figure 6.5

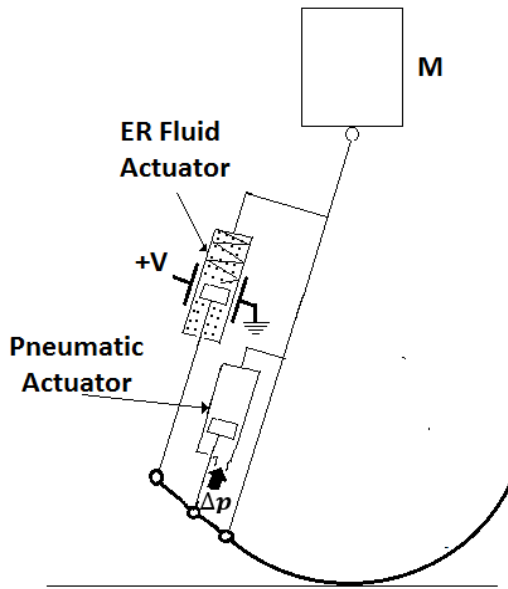


Figure 6.5: mechanical structure of leg for jumping robot

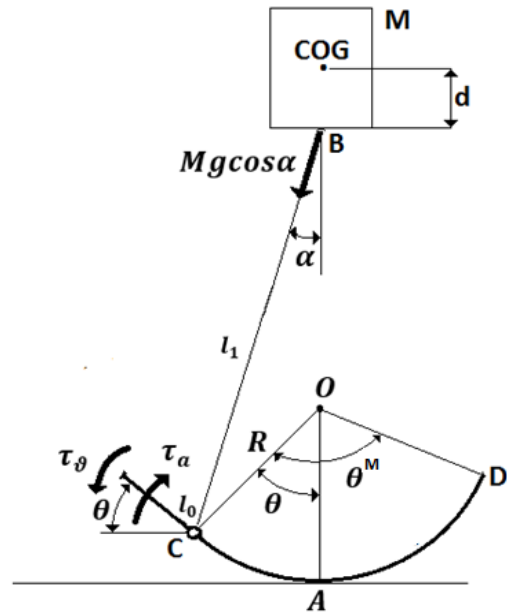


Figure 6.6: mathematical structure for jumping robot

The dynamic model

$$M(d+l_1)^2 \ddot{\theta} + MgR \sin \theta + 2EI\theta + \tau_g = \tau_a - \tau_g \dot{\theta} \quad (6.1)$$

The initial condition of (6.1) is

$$\theta(0) = \theta_0 \quad (6.2)$$

Or

$$M(d+l_1)^2 \ddot{\theta} + J\ddot{\theta} + \mathcal{G}_0\dot{\theta} + K_e\theta = \tau_a - \tau_g \quad (6.3)$$

where the equivalent elastic coefficient of the foot is defined as (when regarding

$$\sin \theta \approx \theta$$

$$K_e = MgR + 2EI \quad (6.4)$$

and τ_g denotes the equivalent torque of the damper (the effect of the ER fluid and local).

6.2. Stance Phase: Touch-Down Sequence

Quality of motion in jumping period is important. The conventional feet discussed in [93] are based by passive dampers which consist of springs and mechanical elastic components. When the leg hits the ground, the damping force increases determining vibrations in mechanical structure that can disturb the evolution of the robot. It is necessary to select appropriate equivalent stiffness and damping coefficients to achieve the desired performances. For this reason, an active damper with ER fluids actuator and a skyhook viscosity controller is proposed (in Figure 6.5). The touch-down sequence starts at the instant when the elastic foot hits the ground. The main instant sequence is presented in Figure 6.7.

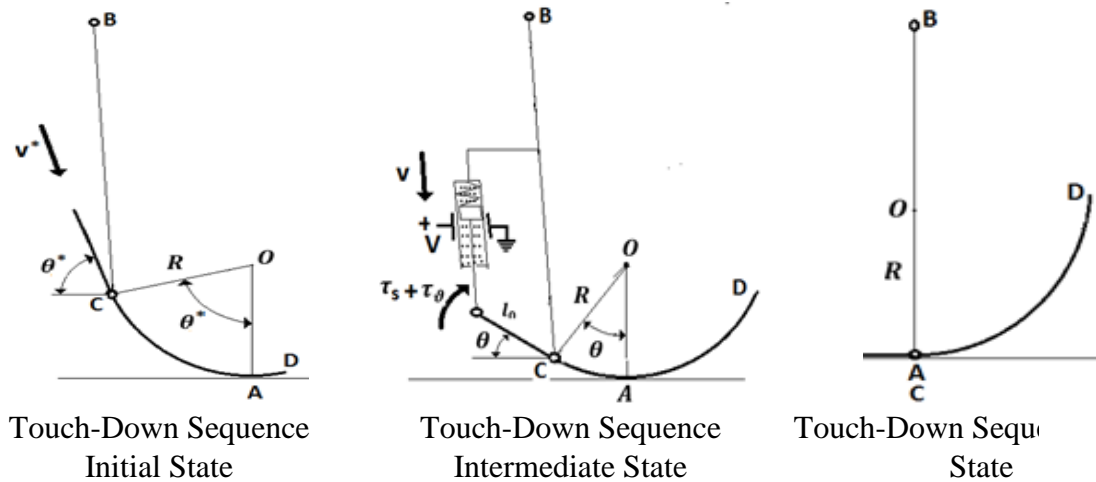


Figure 6.7: Touch-Down Sequence

Case 1: Actuator as passive damper system

The Touch-Down model is illustrated in Figure 6.8 where the spring K_f , K_s denotes the equivalent spring coefficient of elastic foot and elastic parameters, respectively, of the actuator that operates as a semi-active damper system. In this case, we assume that the active torque which is developed by actuator is zero ($\tau_a = 0$), and the actuator operates as a damper, only. This system varies the damping forces using a feedback determined by the dynamics of masses.

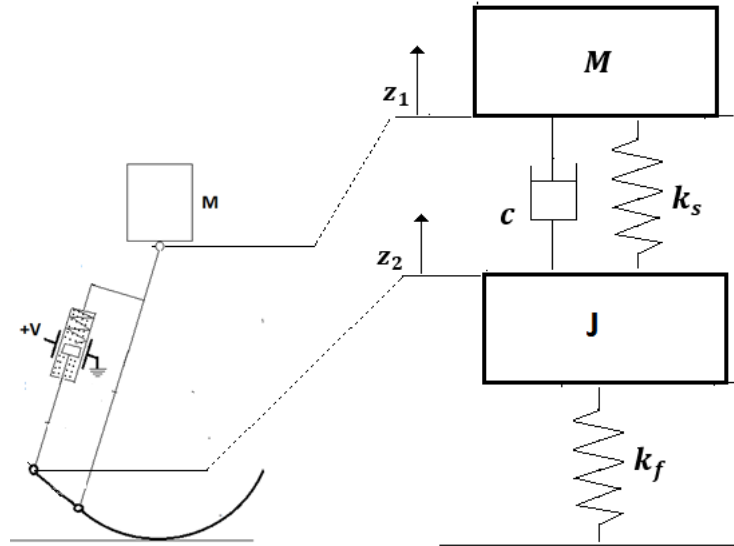


Figure 6.8: Touch-Down passive damper model

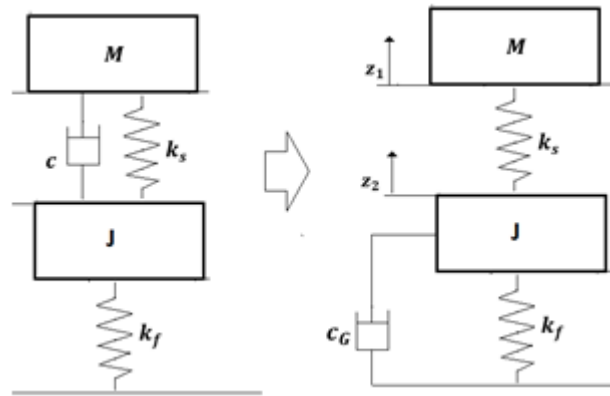


Figure 6.9: Ground-hook damper model

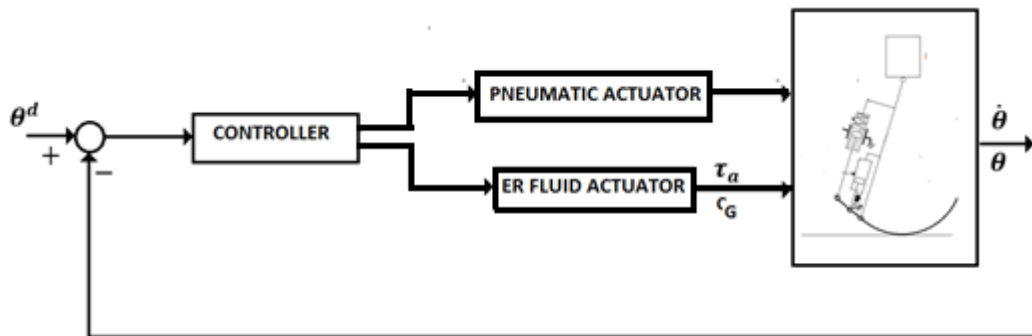


Figure 6.10: Touch-Down sequence control system

The dynamic behaviour of the passive model (in Figure 6.8) is described by using the same procedure as which was discussed in previous section.

$$\ddot{\theta}J = -g_0\dot{\theta} - MgR\sin\theta - 2EI\theta + K_S(z_1 - z_2)R^* + c(\dot{z}_1 - \dot{z}_2) \quad (6.5)$$

where z_1, z_2 represent the vertical positions of the two links of leg (B, C); K_S and

R^* are the elastic coefficient of damper spring and equivalent radius of the damper motion.

$$R^* = l_0 \cos \theta + R \sin \theta \quad (6.6)$$

where c is the passive damping coefficient.

The quality of the system can be evaluated by the analysis of the transmissibility that is defined as in [96].

$$T(\omega) = z_2(\omega)/z_1(\omega) \quad (6.7)$$

Where

$$z_1 = l_0 \sin \theta + R(1 - \cos \theta) + l_1 \cos \alpha \quad (6.8)$$

$$z_2 = l_0 \sin \theta + R(1 - \cos \theta) \quad (6.9)$$

The analyse of the transmissibility of the model which is described in (6.5) is complex due to nonlinear structure. Assuming that oscillations around equilibrium point A is small and the following constraint is verified

$$\left| \frac{R\theta}{l_0} \right| < 1 \quad (6.10)$$

Then

$$z_2 \approx l_0 \theta \quad (6.11)$$

$$R^* \approx l_0 \quad (6.12)$$

Substituting (6.11) and (6.12) into (6.5), we obtain

$$\ddot{z}_2 = -\frac{\mathcal{G}_0 + c_G}{J} \dot{z}_2 - \frac{MgR + 2EI + K_s l_0^2}{J} z_2 + \frac{K_s l_0^2}{J} z_1 + \frac{c l_0^2}{J} \dot{z}_1 \quad (6.13)$$

Applying the Laplace transform, it yields

$$T(s) = \frac{z_2(s)}{z_1(s)} = \frac{K_s l_0^2 + \frac{c l_0^2}{J} s}{s^2 + \frac{\mathcal{G}_0 + c_G}{J} s + \frac{MgR + 2EI + K_s l_0^2}{J}} \quad (6.14)$$

Substitute $s = j\omega$ into (6.14), we obtain

$$T(j\omega) = \frac{z_2(j\omega)}{z_1(j\omega)} = \frac{1 + 2\zeta(j\omega/\omega_n)}{-1 + (\omega/\omega_n)^2 + 2\zeta(\omega/\omega_n)j} \quad (6.15)$$

where ω_n is natural frequency of the system which is calculated as below

$$\omega_n = \sqrt{\frac{MgR + 2EI + K_s l_0^2}{J}} \quad (6.16)$$

and ζ is the equivalent passive damping factor ratios which is calculated as below

$$\zeta_p = \frac{\mathcal{G}_0 + c}{2\sqrt{J(MgR + 2EI + K_s l_0^2)}} \quad (6.17)$$

Case 2: Actuator as semiactive damper system (ground system)

A ‘‘Groundhook’’ strategy (in Figure 6.9) is proposed to facilitate the study of vibrations and to find control solutions to reduce vertical oscillations. A fictitious damper with equivalent viscosity coefficient c_G is considered between body and ground

$$c_G = \begin{cases} c_{\max} \dot{z}_2 & \text{if } -\dot{z}_2 (\dot{z}_1 - \dot{z}_2) > 0 \\ c_{\min} \dot{z}_2 & \text{if } -\dot{z}_2 (\dot{z}_1 - \dot{z}_2) < 0 \end{cases} \quad (6.18)$$

A similar procedure results are shown in (6.19) below

$$\ddot{z}_2 = -\frac{\mathcal{G}_0 + c_G}{J} \dot{z}_2 - \frac{MgR + 2EI + K_s l_0^2}{J} z_2 + \frac{K_s l_0^2}{J} z_1 \quad (6.19)$$

which leads to

$$T(s) = \frac{z_2(s)}{z_1(s)} = \frac{K_s l_0^2}{s^2 + \frac{\mathcal{G}_0 + c_G}{J} s + \frac{MgR + 2EI + K_s l_0^2}{J}} \quad (6.20)$$

$$T(j\omega) = \frac{z_2(j\omega)}{z_1(j\omega)} = \frac{\eta}{-1 + \left(\frac{\omega}{\omega_n}\right)^2 + 2\zeta_G \frac{\omega}{\omega_n} j} \quad (6.21)$$

where ω_n is natural frequency of system and shown in (6.16) and ζ_G is the equivalent damping factor ratios

$$\zeta_G = \frac{\mathcal{G}_0 + c_G}{2\sqrt{J(MgR + 2EI + K_s l_0^2)}} \quad (6.22)$$

$$\eta = \frac{K_s l_0^2}{MgR + 2EI + K_s l_0^2} \quad (6.23)$$

Case 3: Actuator as ER Driver System

The actuator operates as ER driver system that develops an active torque τ_a

$$\ddot{\theta} J = -\mathcal{G}_0 \dot{\theta} - MgR \sin \theta - 2EI \theta + K_s (z_1 - z_2) R^* + c_g (\dot{z}_1 - \dot{z}_2) + \tau_a \quad (6.24)$$

where c_g is non-active value of the ER damping coefficient. Considering the constraint in (6.10), this model can be written as

$$\ddot{\theta} = -\frac{g_0 + c_g l_0^2}{J} \dot{\theta} - \frac{MgR + 2EI + K_s l_0^2}{J} \theta + \frac{K_s l_0}{J} z_1 + \frac{c_g l_0}{J} \dot{z}_1 + \frac{1}{J} \tau_a \quad (6.25)$$

New state variables are defined as

$$\begin{cases} x = [x_1 & x_2]^T = [\theta & \dot{\theta}]^T \\ z = [z_1 & z_2]^T \end{cases} \quad (6.26)$$

where z denotes the disturbance variable that modify the system behaviour.

The dynamic model becomes

$$\dot{x} = Ax + b\tau_a + Dz \quad (6.27)$$

$$y = c^T x \quad (6.28)$$

where

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{MgR + 2EI + K_s l_0^2}{J} & -\frac{g_0 + c_g l_0^2}{J} \end{bmatrix}; \quad b = \begin{bmatrix} 0 \\ \frac{1}{J} \end{bmatrix}; \quad D = \begin{bmatrix} 0 & 0 \\ \frac{K_s l_0}{J} & \frac{c_g l_0}{J} \end{bmatrix}$$

The matrix A is stable but the stability, which describes system performance, is worsen by the disturbance variable z . This disturbance can be evaluated in terms of state variables as

$$z_1 = \alpha \theta \quad (\alpha < \alpha^*) \quad (6.30)$$

$$z_2 = \beta \dot{\theta} \quad (\beta < \beta^*) \quad (6.31)$$

where α , β are positive constants. Therefore, the dynamic model in (6.27) becomes

$$\dot{x} = A^* x + b\tau_a \quad (6.32)$$

Where

$$A^* = \begin{bmatrix} 0 & 1 \\ -\frac{MgR + 2EI + K_s l_0^2}{J} + \alpha & -\frac{g_0 + c_g l_0^2}{J} + \beta \end{bmatrix} \quad (6.33)$$

Clearly, the disturbances z determine the instability of the system (A^* is an unstable matrix). The control law is proposed as

$$\tau_a = -ky \quad (6.34)$$

Where $k = const > 0$ satisfies the sector condition below

$$k_{\min} \leq k \leq k_{\max} \quad (6.35)$$

Theorem 6.1: The state vector $[x_1 \ x_2]^T = [\theta \ \dot{\theta}]^T$ converges toward zero if the following conditions are satisfied

a) Matrix $H = A^* - E$ is Hurwitz, where $E = ec^T$ is a symmetrical matrix.

b) (H, b) is controllable and (H, c) is observable.

$$c) \operatorname{Re} \left\{ \frac{c^T}{2} (sI - H)^{-1} (b - ek^{-1}) \right\} + k^{-1} \geq 0$$

Proof:

Selecting Lyapunov function as

$$V = \frac{x^T P x}{2}$$

where P is a symmetrical positive definite matrix. Derivating (6.37) by time and using (6.32), we obtain

$$\dot{V} = x^T \left[(A^*)^T + P A^* \right] x + 2x^T P b u \quad (6.38)$$

Or

$$\dot{V} = x^T \left\{ \left[(A^*)^T - E \right] P + P (A^* - E) \right\} x + 2x^T P E u + 2x^T P b u \quad (6.39)$$

Considering $E = wc^T$ and the control law in (6.34), the last two terms can be written as

$$2x^T P E u + 2x^T P b u = 2x^T P (ex + bu) = 2x^T P \left(b - \frac{e}{k} \right) u \quad (6.40)$$

From (6.35), after some calculations, it yields

$$\dot{V} \leq x^T \left\{ \left[(A^*)^T - E \right] P + P (A^* - E) \right\} x + 2x^T \left[P \left(b - \frac{e}{k} \right) - \frac{c^T}{2} \right] u - \frac{u^2}{k} \quad (6.41)$$

By using the Yakubovich-Kalman-Popov Lemma [98] and conditions a, b, c of Theorem 6.1, it yields

$$\left[(A^*)^T - E \right] P + P (A^* - E) = -qq^T \quad (6.42)$$

$$P \left(b - \frac{e}{k} \right) - \frac{c^T}{2} = q\sqrt{k^{-1}} \quad (6.43)$$

Substituting (6.42), (6.43) into (6.41), we obtain

$$\dot{V} \leq -\left(x^T q - u\sqrt{k^{-1}}\right)^2 < 0 \quad (6.44)$$

Remark 6.1:

Define the transfer function $G(s)$ as follows

$$G(s) = \frac{c^T}{2}(sI - H)^{-1}(b - ek^{-1}) \quad (6.45)$$

Considering the inequality (6.35) and condition c) of the Theorem 6.1 can be re-written as circle criterion [98]

$$\operatorname{Re}\left(\frac{k_{\max}^{-1} + G(j\omega)}{k_{\min}^{-1} + G(j\omega)}\right) > 0 \quad (6.46)$$

6.3. Stance Phase: Take-off Sequence

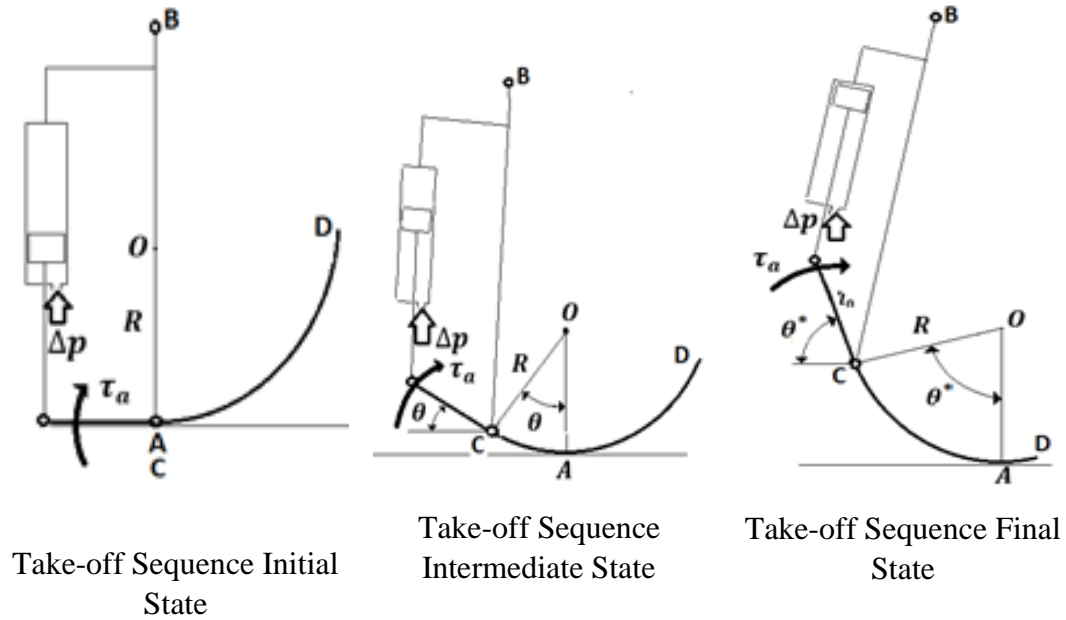


Figure 6.11: Take-off Sequence

During this sequence, actuator system has to develop a sufficiently large energetic pulse to ensure a jumping motion on specified trajectory. This condition can be synthesized as

$$W(t) \geq W^* \quad (6.47)$$

$$\frac{dW(t)}{dt} \geq \gamma > 0 \quad (6.48)$$

where $W(0) = 0$ and W^* is critical energy that satisfies the trajectory parameters and $\gamma = \text{const} > 0$ determined by the take-off impulse.

Define v^* the starting velocity on the flight trajectory for the position $\theta = \theta^*$ (in Figure 6.11c). The critical energy will be

$$W^* = \frac{1}{2} K_f (\theta^*)^2 + \frac{1}{2I_0^2} J (\dot{\theta}^*)^2 \quad (6.49)$$

Total active energy can be expressed as

$$W = w^T w = \frac{1}{2} K_f \theta^2 + \frac{1}{2} J \dot{\theta}^2 \quad (6.50)$$

where w is the energy component vector that is calculated as in (6.51) below and K_f is the equivalent elastic coefficient of the foot.

$$w = \begin{bmatrix} \frac{\theta}{\sqrt{\frac{2}{K_e}}} & \frac{\dot{\theta}}{\sqrt{\frac{2}{J}}} \end{bmatrix}^T$$

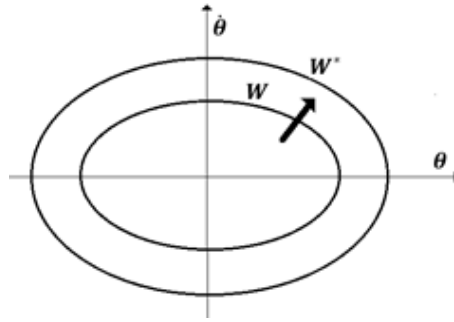


Figure 6.12: energy ellipsoid

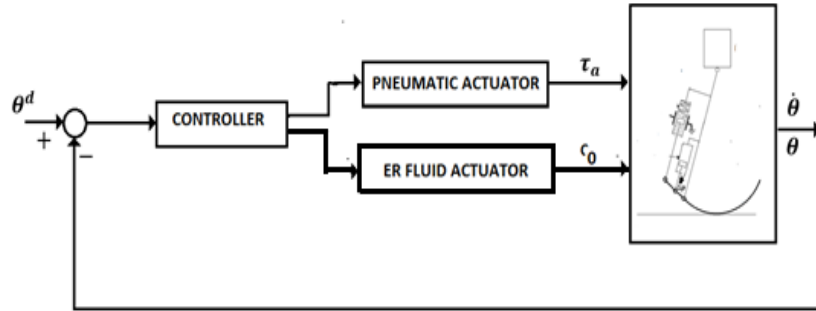


Figure 6.13: Take-off sequence control system

The constraint (6.47) is shown in the ellipsoid energy (in Figure 6.12) and the control system (in Figure 6.13) ensures the jumping conditions.

Theorem 6.2: The jumping conditions (6.47) and (6.48) are satisfied if the control law has the form below

$$\tau_a = -k_1^J \theta + k_2^J \dot{\theta} \quad (6.52)$$

where k_1^J , k_2^J are the controller gains, positive constants, that satisfy following

conditions:

$$k_1^J > K_f - K_e \quad (6.53)$$

$$2k_2^J - k_1^J > 2(\mathcal{G}_0 + c_0) - K_f + K_e \quad (6.54)$$

Proof

Dynamic model can be inferred from (6.1)-(6.3) where the damper is considered as a passive damper with the damping coefficient c_0 and the actuator operates as a pneumatic system that develops an active torque τ_a

$$\ddot{\theta}J = -(\mathcal{G}_0 + c_0)\dot{\theta} - (MgR + 2EI)\theta + \tau_a \quad (6.55)$$

Where

$$\tau_a = \Delta p S l_0 \quad (6.56)$$

$\Delta p = p_f - p_0$ is the expression variation in the actuator and S is the area of piston surface.

The derivative of (6.49) will be

$$\dot{W} = K_f \theta \dot{\theta} + J \dot{\theta} \ddot{\theta} \quad (6.57)$$

Substituting the dynamic model in (6.53), it yields

$$\dot{W} = K_f \theta \dot{\theta} + \dot{\theta} \left[-(\mathcal{G}_0 + c_0)\dot{\theta} - K_e \theta + \tau_a \right] \quad (6.58)$$

Substitute the control law in (6.52) into (6.58), we obtain

$$\dot{W} = (K_f - K_e - k_1^J) \theta \dot{\theta} + \left[k_2^J - (\mathcal{G}_0 + c_0) \right] \dot{\theta}^2 \quad (6.59)$$

Then, by applying the inequality

$$\theta \dot{\theta} \geq -\frac{\theta^2}{2} - \frac{\dot{\theta}^2}{2} \quad (6.60)$$

(6.58) becomes

$$\dot{W} \geq \sigma_1 \frac{\theta^2}{2} + \sigma_2 \frac{\dot{\theta}^2}{2} \quad (6.61)$$

Where

$$\sigma_1 = k_1^J - K_f + K_e \quad (6.62)$$

$$\sigma_2 = 2k_2^J - k_1^J - 2(\mathcal{G}_0 + c_0) + K_f - K_e \quad (6.63)$$

From (6.53), (6.54), it yield $\sigma_1 > 0$, $\sigma_2 > 0$. Combining with (6.61), we obtain

$$\dot{W} > \gamma > 0 \quad (6.64)$$

Considering the following relations

$$\rho = 2\frac{\sigma_1}{K_f}; \quad \sigma_2 = \rho\frac{J}{2} + \sigma_3 \quad (6.65)$$

where $\sigma_3 = \text{const} > 0$, then, \dot{W} can be written as

$$\dot{W} = \rho\frac{K_f}{2}\dot{\theta}^2 + \left(\rho\frac{J}{2} + \sigma_3\right)\dot{\theta}^2 > \rho\left(\frac{1}{2}K_f\dot{\theta}^2 + \frac{1}{2}J\dot{\theta}^2\right) = \rho W \quad (6.66)$$

From (6.64) and (6.66), it can be concluded that W is an increasing positive definite function. For the final position of $\theta = \theta^*$, the limit value of $W = W^*$ is achieved.

Chapter 7: SIMULATION OF CONTROL ALGORITHMS

7.1. LQR Control Simulation of E-IP Model.

Consider the E-IP model (fig 5.3) and a LQR controller where the matrix R is assumed as identity matrix and the components of matrix Q are selected through GA. Simulating system under LQR controller in 10s with sample time as 10ms, there are 1001 samples of system response in a period of simulation time.

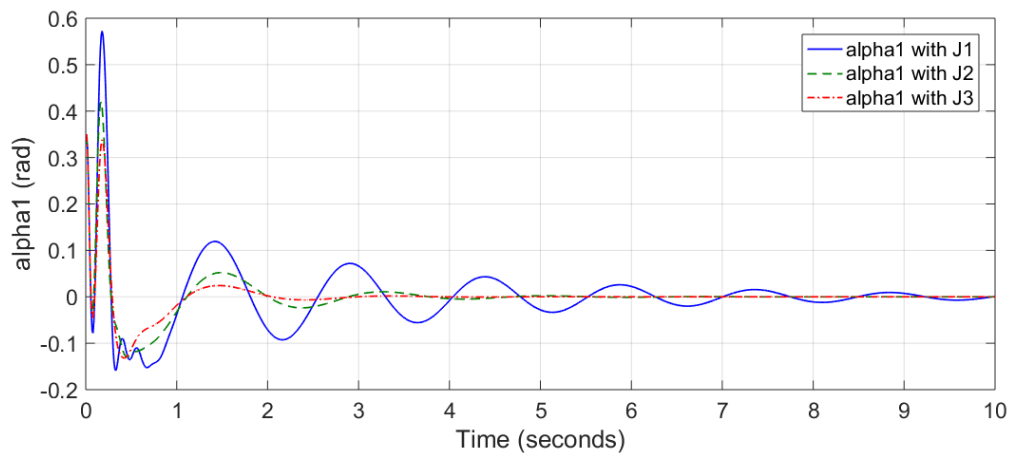


Figure 7.1: Comparison among responses of E-IP under LQR controllers through α_1 (rad)

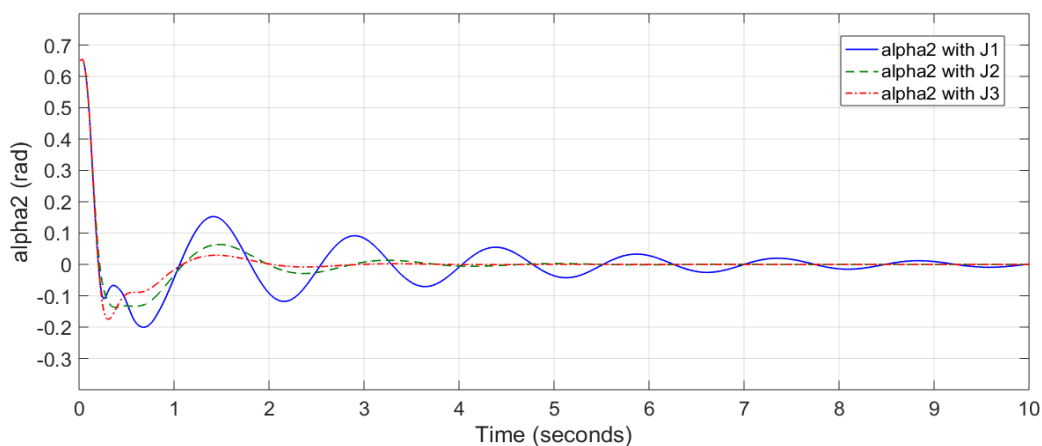


Figure 7.2: Comparison among responses of E-IP under LQR controllers through α_2 (rad)

7.2. HSM Control for E-IP System.

Consider a HSM control where the control signal is

$$u = \frac{(a_1 a_2 g_1 u_{eq1} + a_2 g_2 u_{eq2} + g_3 u_{eq3}) - (k_3 S_3 + \eta_3 \text{sign} S_3)}{a_1 a_2 g_1 + a_2 g_2 + g_3} \quad (7.1)$$

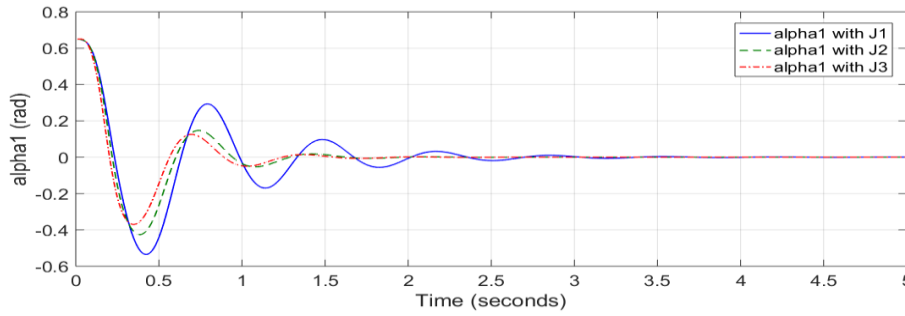


Figure 7.3: Comparison among responses of E-IP under HSM controllers through α_1 (rad)

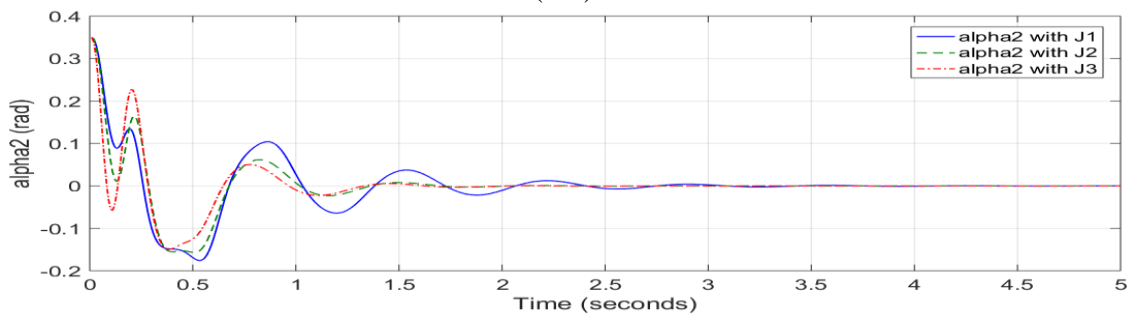


Figure 7.4: Comparison among responses of E-IP under HSM controllers through α_2 (rad)

7.3. Conventional PD Control for Two-Legged Robot.

Consider a conventional PD controllers for the motion control of AR robot.

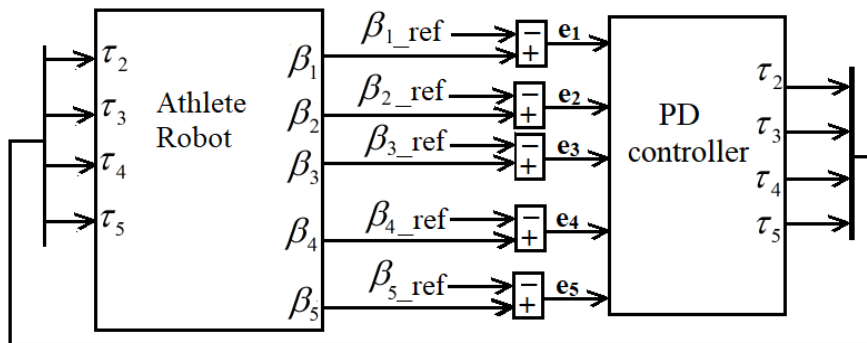


Figure 7.5: Structure of PID controller for step motion of AR

Simulation results of step-motion under PID controller which is described in Figure 7.5 are listed as below.

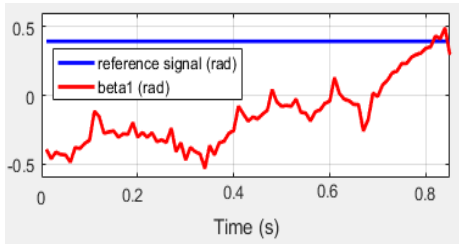


Figure 7.6: Reference signal β_{1_ref} and β_1

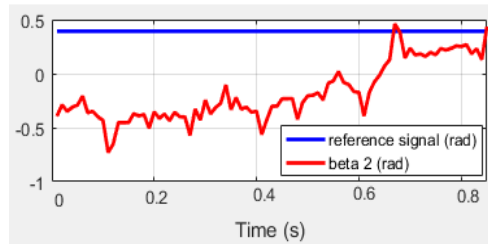


Figure 7.7: Reference signal β_{2_ref} and β_2

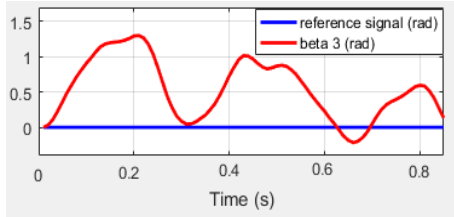


Figure 7.8: Reference signal β_{3_ref} and β_3

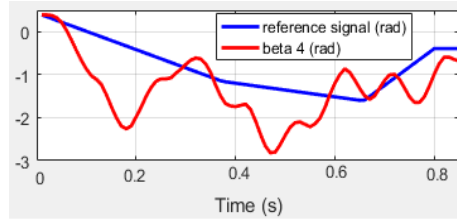


Figure 7.9: Reference signal β_{4_ref} and β_4

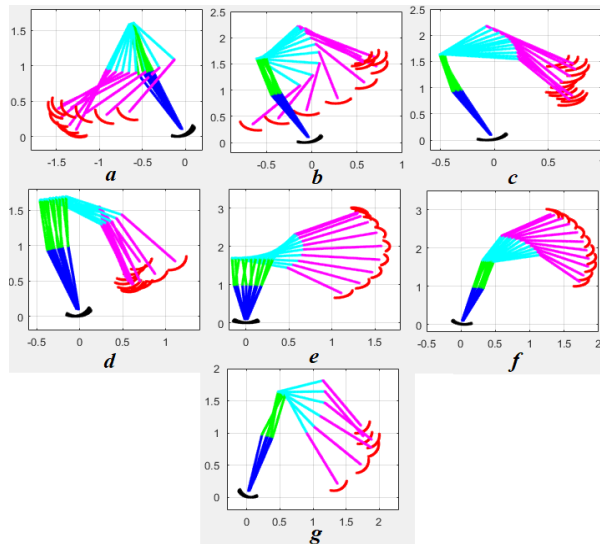
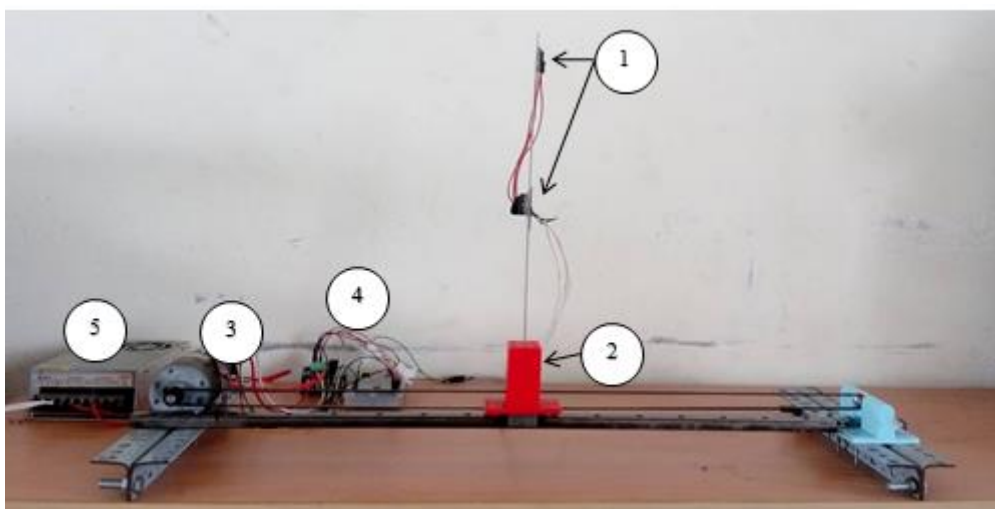


Figure 7.10: Motion of AR

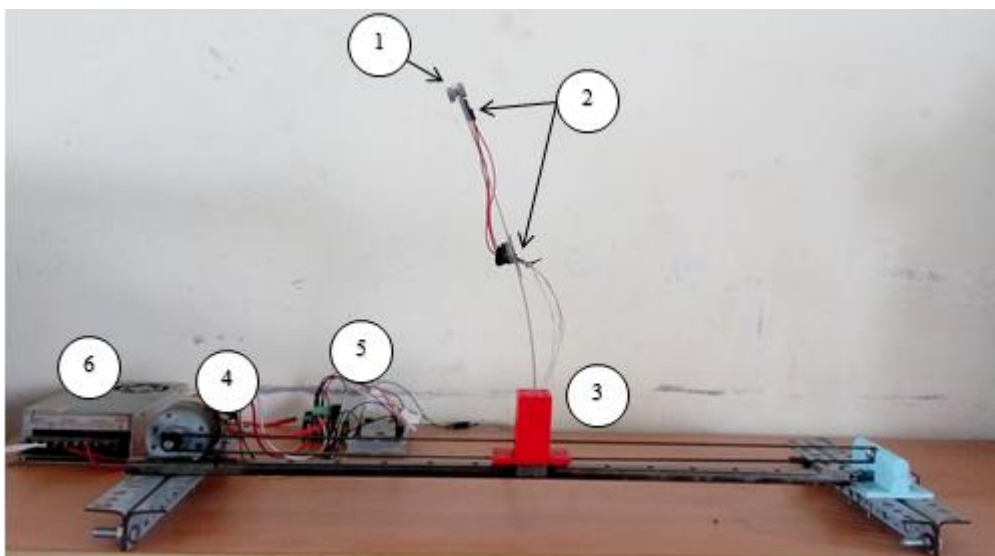
Chapter 8: EXPERIMENTAL STUDY OF MODELS WITH ELASTIC COMPONENTS

8.1. Elastic Inverted Pendulum

An experimental E-IP platform is presented in Figure 8.1 below. According to mathematical model in Figure 5.1, one MPU sensor is located at the middle of the elastic beam to measure angle α_2 . Another MPU sensor is located at the top of the elastic beam to measure angle α_1



(a)



(b)

Figure 8.1: Experimental model of E-IP

8.2. Two-legged robot with Elastic legs

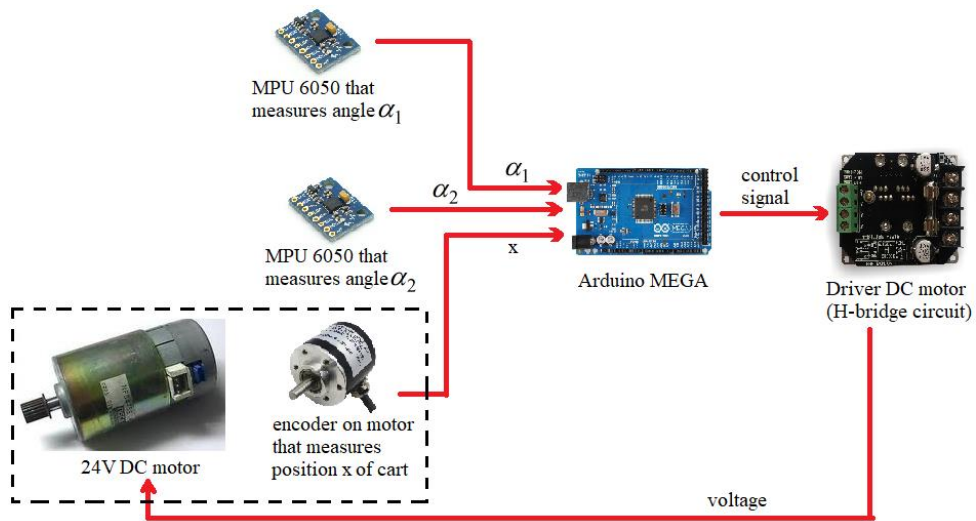


Figure 8.2: Electronics structure for E-IP model

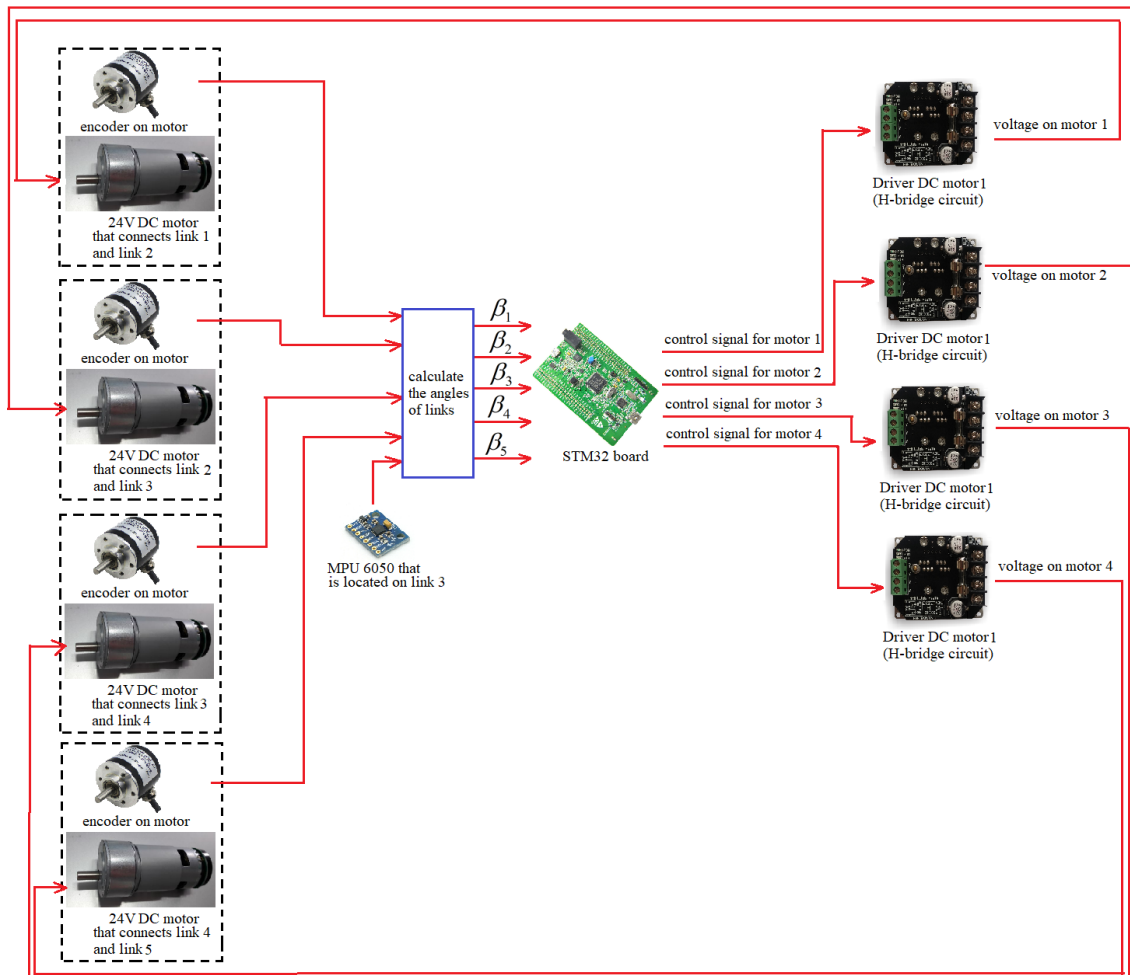


Figure 8.3: Experimental structure of AR hardware

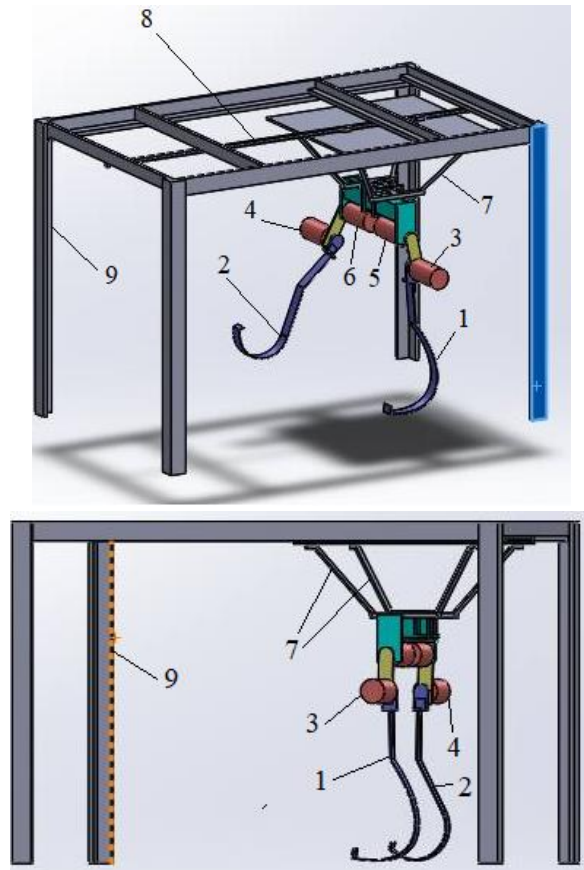


Figure 8.4: Experimental model of AR in Solidworks description

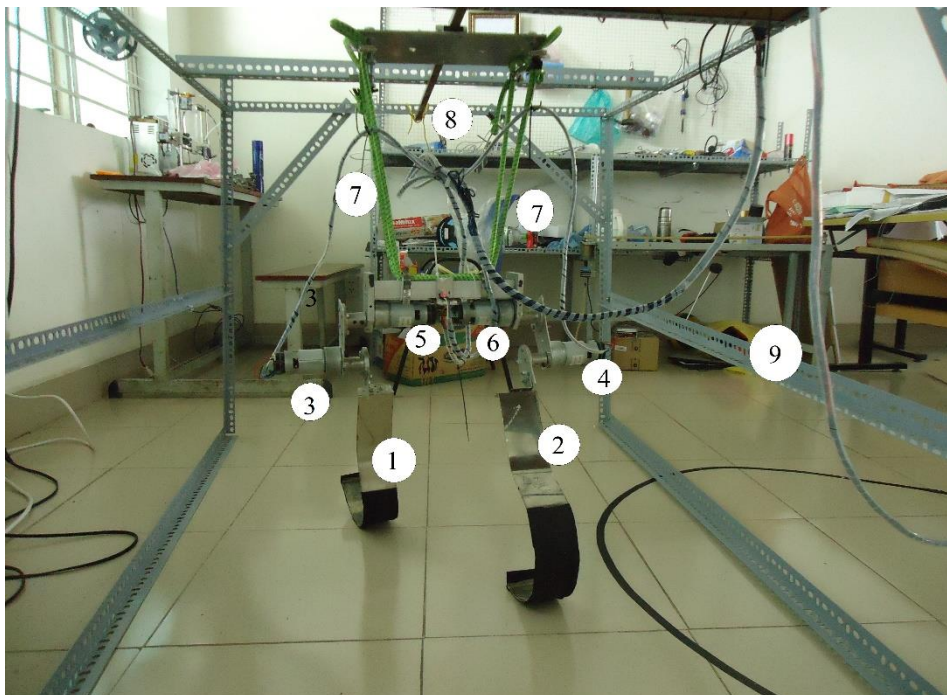


Figure 8.5: Real experimental robot in behind direction

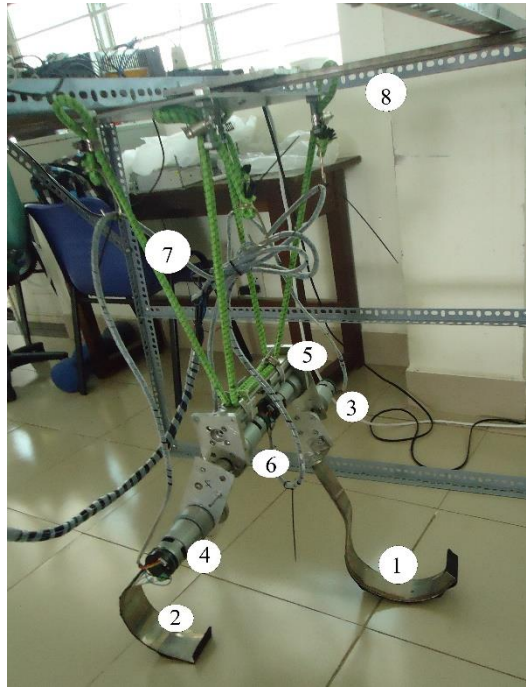
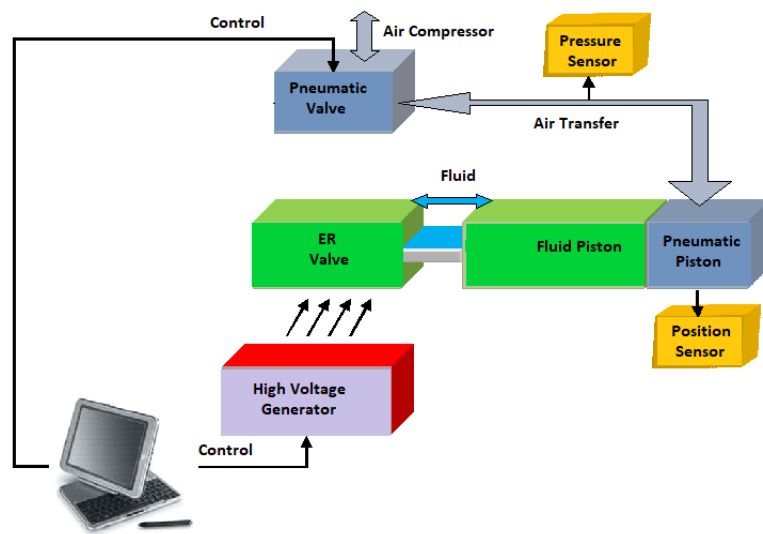


Figure 8.6: Real experimental robot in crossover direction



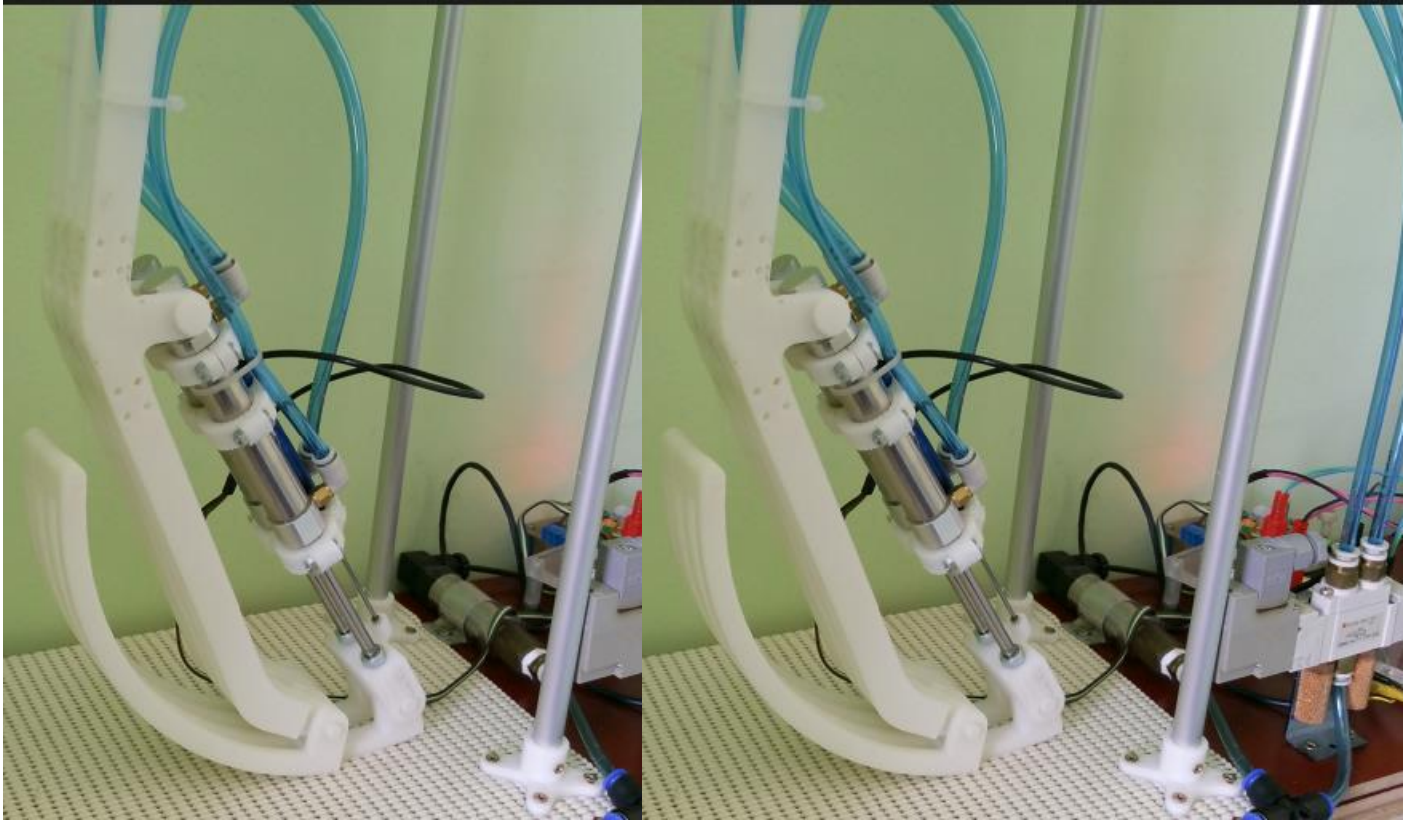


Figure 8.7: Experimental platform- mechanical architecture (Photos)

REFERENCE

- [1] Kent H. Lundberg, Taylor W. Barton, “History of Inverted-Pendulum Systems”, IFAC Proceedings Volumes, Vol. 42, Issue. 24, pp. 131-135, Elsevier, 2010.
- [2] Furuta, K., Yamakita, M. and Kobayashi, S, “Swing-up control of IP using pseudo-state feedback”, Journal of Systems and Control Engineering, 206(6), 263-269, 1992.
- [3] Andrew Careaga Houck, Robert Kevin Katschmann, Joao Luiz Almeida Souza Ramos, “Furuta Pendulum”, Project of Advances System Dynamics & Control, Department of Mechanical Engineering, Massachusetts Institute of Techonology, Fall 2013.
- [4] Olfar Boubaker, “The IP: a fundamental Benchmark in Control Theory and Robotics”, pp 1-6, International Conference on Education and e-Learning Innovations (ICEELI), IEEE, 2012.
- [5] Vo Anh Khoa, Nguyen Minh Tam, Le Thi Thanh Hoang, Nguyen Thien Van, **Nguyen Van Dong Hai**, “Model and Control Algorithm Construction for Rotary Inverted Pendulum in Laboratory”, Journal of Technical Education and Science, ISSN: 18959-1272, 2018. (in Vietnamese) (accepted)
- [6] US patent 5,701,965 Human transporter
- [7] US Patent 6,302,230 Personal mobility vehicles and methods
- [8] US patent 6,616,313 Motorized transport vehicle for a pedestrian

- [9] Two-wheel, self-balancing vehicle with independently movable foot placement sections US 8738278 B2
- [10] [Patent USD739307 - One-wheeled vehicle](#)
- [11] R. Sethunadh, P. P. Mohanlal, “Virtual instrument based dynamic balancing system for rockets and payloads”, Autotestcon, IEEE, pp. 291-296, 2007. DOI: [10.1109/AUTEST.2007.4374232](#)
- [12] Shuai Sun, Zhishan Zhang, Quan Pan, Cangan Sun, “Controller design for anti-heeling system in container ships”, 35th Chinese Control Conference (CCC), pp. 5798-5803, IEEE, 2016. DOI: [10.1109/ChiCC.2016.7554263](#)
- [13] Pencheng Wang, Zhihong Man, Zhenwei Cao, Jinchuan Zheng, Yong Zhao, “Dynamics modelling and linear control of quadcopter”, International Conference on Advanced Mechatronic Systems (ICAMechS), IEEE, 2016.
- [14] Tran Vi Do, Ho Trong Nguyen, Nguyen Minh Tam, **Nguyen Van Dong Hai**, “Balancing Control for Double-linked IP on Cart: Simulation and Experiment”, Journal of Technical Education Science, ISSN: 1859-127, No. 44A, pp. 68-75, November-2017.
- [15] V. Casanova, J. Salt, R. Piza, A. Cuenca, “Controlling the Double Rotary inverted pendulum with Multiple Feedback Delays”, Vol. VII, No. 1, pp. 20-38, 2012.
- [16] **Nguyen Van Dong Hai**, Nguyen Phong Luu, Nguyen Minh Tam, Hoang Ngoc Van, “Optimal Control for Quadruped inverted pendulum”, pp. 18-23, Vol. 34, Journal of Technical Education Science, Vietnam, ISSN: 1859-1272, 2016.
- [17] A. Gmiterko, M. Grossman, “N-link Inverted Pendulum Modelling”, Recent Advances in Mechatronics, pp. 151-156, Recent Advances in Mechatronics, Springer 2017.
- [18] Tran Hoang Chinh, Nguyen Minh Tam, **Nguyen Van Dong Hai**, “A Method of PID-FUZZY control for pendubot”, Journal of Technical Education Science, No. 44A, pp. 61-67, ISSN: 1859-127, November-2017.
- [19] Huynh Xuan Dung, Huynh Duong Khanh Linh, Vu Dinh Dat, Nguyen Thanh Phuong, Nguyen Minh Tam, **Nguyen Van Dong Hai**, “Application of Fuzzy Algorithm in Optimizing Hierarchical SMC for Pendubot System”, International Journal of Robotica & Management, Vol. 22, Nr. 2, Dec-2017.
- [20] Scott C. Brown, Kevin M. Passino, “Intelligent Control for an Acrobot”, Journal of Intelligent and Robotic Systems, No. 18, pp. 209-248, Netherlands, 1999.
- [21] Umashankar Nagarajan, George Kantor, Ralph Hollis, “The ballbot: An omnidirectional balancing mobile robot”, The International Journal of Robotics Research, 2013.
- [22] Nguyen Minh Tam, **Nguyen Van Dong Hai**, Nguyen Phong Luu, Le Van Tuan, “Modelling and Optimal Control for Two-wheeled Self-Balancing Robot”, Journal of Technical Education Science, Vietnam, ISSN: 1859-1272, Vol. 37, pp. 35-41, 2016.

- [23] Nguyen Minh Hoang, Ngo Van Thuyen, Nguyen Minh Tam, Le Thi Thanh Hoang, **Nguyen Van Dong Hai**, “Desiging Linear Feedback Controller for E-IP with Tip Mass”, *International Journal of Robotica & Management*, pp. 27-32, Vol. 21, Nr. 2, December-2016.
- [24] Toshiyuki Hayase, Yoshikazu Suematsu, “Control of a flexible IP”, Vol. 8, Issue. 1, *Journal of Advanced Robotics*, pp. 1-12, Taylor& Francis, 1993.
- [25] Andrzej Kot, Agata Nawrocka, “Modeling of Human Balance as an IP”, 15th *International Carpathian Control Conference (ICCC)*, pp. 254-257, IEEE, 2014.
- [26] Akihiro Sato, “A Planar Hopping Robot with One Actuator: Design, Simulation, and Experimental Results”, Master thesis, McGill University, Canada, 2004.
- [27] Ismail Uyamk, “Adaptive Control of a One-legged Hopping Robot through Dynamically Embedded Spring Loaded IP”, Master thesis, Bilkent University, 2011.
- [28] Patrick M. Wensing and David E. Orin, “Control of Humanoid Hopping Based on a SLIP Model”, *Advances in Mechanisms, Robotics and Design Education and Research*, Part of the *Mechanisms and Machine Science* book series, pp. 265-274, Springer, 2013.
- [29] Full, R. J. and Koditschek, D. E., “Templates and anchors: neuromechanical hypotheses of legged locomotion on land”, *Journal of Experimental Biology*, 202(23):3325–3332, 1999.
- [30] J. G. Ketelaar, L. C. Visser, S. Stramigioli and R. Carloni, “Controller Design for a Bipedal Walking Robot using Variable Stiffness Actuators”, *IEEE International Conference on Robotics and Automation (ICRA)*, pp. 5650-5655, IEEE, Germany, May-2013.
- [31] Yiping Liu, “A Dual-SLIP Model for Dynamic Walking in a Humanoid over Uneven Terrain”, PhD thesis, Ohio State University, 2015.
- [32] Yiping Liu, Patrick M. Wensing, James P. Schmiedeler, and David E. Orin, “Terrain-Blind Humanoid Walking Based on a 3D Actuated Dual-SLIP Model”, *IEEE Robotics and Automation Letters*, 2016.
- [33] Mathew D. Berkerneier, Kamal V. Desai, “Design of a Robot Leg with Elastic Energy Storage, Comparison to Biology, adn Preliminary Experimental Results”, pp. 213-218, *Proceedings of the 1996 IEEE International Conference on Robotics and Automatio*, Minneapolis, 2016.
- [34] Jerry E. Pratt, Benjamin T. Krupp, “Series Elastic Actuators for legged robots”, *Proceedings of SPIE-The International Society for Optical Engineering*, 2004. DOI:10.1117/12.548000
- [35] Fantoni, I., Lozano, R., “Nonlinear Control for Under-actuated Mechanical System”, Springer-Verlag, London, 2002.
- [36] Ho Trong Nguyen, Nguyen Minh Tam, **Nguyen Van Dong Hai**, “Application of Genetic Algorithm in Optimization Controller for Cart and Pole System”, *Journal of Technical Education Science*, ISSN: 1859-127, No. 44A, pp. 41-47, November, 2017.

- [37] Nguyen Van Dong Hai, “Input-output Linearization controller for Cart and Pole system”, Master Thesis of Automation and Control, Ho Chi Minh city University of Technology (HCMUT), Vietnam, 2011.
- [38] Beletzky V.V, “Nonlinear Effects in Dynamics of Controlled Two-legged Walking”, Part of Book of Nonlinear Dynamics in Engineering Systems, International Union of Theoretical and Applied Mechanics, pp. 17-26, Springer, 1990.
- [39] Sujan, W., Amin, B., Nathan, S. & Madhavan, S, “Bipedal Walking – A Developmental Design”, In Proceedings of International Symposium on Robotics and Intelligent Sensors, Procedia 41, pp. 1016-1021, Elsevier, 2012.
- [40] Qinghua, L., Takanishi, A. & Kato, I, “A Biped Walking Robot having a ZMP Measurement System using Universal Force-moment Sensors”, In *Proceeding of Intelligent Robots and System*, pp. 1568-1573, IEEE, 1991.
- [41] Kim, D. W., Kim, N. H. & Park, G. T, “ZMP based Neuron Network Inspired Humanoid Robot Control”, *Journal of Nonlinear Dynamics*, 67(1), pp. 793-806. Springer, 2012.
- [42] J. A. Smith & A. Seyfarth, “Elastic Leg Function in a Bipedal Walking Robot”, *Journal of Biomechanics*, Page S306, Vol. 40, Supplement 2, Elsevier, 2007.
- [43] Maziar Ahmad Sharbafi, Christian Rode, Stefan Kurowski, Dorian Scholz, Rico Mockel, Katayon Radkhah, Goouping Zhao, Aida Mohammadinejad Rashty, Oskar von Stryk & Andre Seyfarth, “A New Biarticular Actuator Design Facilities Control of Leg Function in *BioBiped3*”, *Journal of Bioinspiration & Biomimetics*, Vol. 11, No. 4, IOP Publishing, 2016.
- [44] Ryuma Niiyama, Satoshi Nishikawa, Yasuo Kuniyoshi, “Biomechanical Approach to Open-Loop Bipedal Running with a Musculoskeletal Athlete Robot”, *Journal of Advanced Robotics*, Vol. 26, Issue. 3-4, 2012.
- [45] Ryuma Niiyama and Yasuo Kuniyoshi, “Design of a Musculoskeletal Athlete Robot: A Biomachanical Approach”, *Proceedings of the Twelfth International Conference on Climbing and Walking Robots and the Support Technologies for Mobile Machines*, Turkey, 2009.
- [46] Gianluca Garofalo, Christian Ott & Alin Albu-Schaffer, “Walking control of fully actuated robots based the Bipedal SLIP model”, In *Proceeding of International Conference on Robotics and Automation (ICRA)*, pp. 1456-1463. IEEE, 2012.
- [47] Mohammad Shabazi, Robert Babuska & Gabriel A. D. Lopes, “Unified Modeling and Control of Walking and Running on IP”, *IEEE Transactions of Robotics*, Vol. 32, Issue 5, pp. 1178-1195, 2016.
- [48] **Nguyen Van Dong Hai**, Mircea Ivanescu, Mircea Nitulescu, “Hierarchical Sliding Mode Control for Balancing Athlete Robot”, 21st International Conference on System Theory, Control and Computing (ICSTCC 2017), Sinaia, Romania, Nov-2017.
- [49] **Nguyen Van Dong Hai**, Huynh Xuan Dung, Nguyen Minh Tam, Cristian Vladu, Mircea Ivanescu, “Hierarchical Sliding Mode Algorithm for Athlete Robot Walking”, *Journal of Robotics*, Article ID 6348980, Hindawi, December-2017. DOI: doi.org/10.1155/2017/6348980 (ISI/ESCI/SCOPUS journal)

- [50] Castigliano A, “Elastic Stresses in Structures”, Cambridge University Press, 2014.
- [51] Endo, G., Morimoto, J., Nakanishi, J. & Cheng, G, “An Empirical Exploration of a Neural Oscillator for Biped Locomotion Control”, In *Proceedings of International Conference on Robotics & Automation*, pp. 3036-3042. IEEE, 2004.
- [52] Hein, D., Hild, M. & Berger, R, “Evolution of Biped Walking Using Neural Oscillators and Physical Simulation”, *Part of the Lecture Notes in Computer Science book series (LNCS), Vol. 5001*, pp. 433-440. Springer, 2007.
- [53] Lothar M. Schmitt, “Theory of genetic algorithms”, Theoretical Computer Science, Vol. 259, Issues 1-2, pp. 1-61, Elsevier, May-2001.
- [54] Dianwei Qian, Jianqiang Yi, Dongbin Zhao, Yinxing Hao, “Hierarchical Sliding Mode Control for Series Double Inverted Pendulum System”, International Conference on Intelligent Robots and Systems, IEEE, 2006. DOI: [10.1109/IROS.2006.282521](https://doi.org/10.1109/IROS.2006.282521)
- [55] Dianwei Qian, Jianqiang Yi, Dongbin Zhao, “Hierarchical Sliding Mode Control for a Class of SIMO Under-actuated Systems”, Journal Control and Cybernetics, Vol. 37, No. 1, 2008.
- [56] Qian, Dianwei, Yi, Jiangqiang, “Hierarchical SMC for Under-actuated Cranes: Design, Analysis and Simulation”, Book of Control Engineering, Springer-Verlag, 2016.
- [57] Vu Duc Ha, Huynh Xuan Dung, Nguyen Minh Tam, **Nguyen Van Dong Hai**, “Hierarchical Fuzzy Sliding Mode Control for a Class of SIMO Under-actuated Systems”, Journal of Technical Education Science, ISSN: 1859-1272, Vietnam, 2017.
- [58] Kamal Rsetam, Zhenwei Cao, Zhihong Man, “Hierarchical Sliding Mode Control applied to a single-link flexible joint robot manipulator”, International Conference on Advanced Mechatronic Systems, IEEE, 2016.
- [59] Vu Dinh Dat, Huynh Xuan Dung, Phan Van Kiem, Nguyen Minh Tam, **Nguyen Van Dong Hai**, “A method of Fuzzy-Sliding Mode Control for Pendubot model”, Journal of Science and Technology-University of Da Nang, Vietnam, ISSN: 1859-1591, No. 11 (120), Issue 1, pp. 12-16, 2017.
- [60] E. Trillas, “Lotfi A. Zadeh: On the man and his work”, [Scientia Iranica, Volume 18, Issue 3](https://doi.org/10.1007/978-94-007-5749-7_3), June 2011, Pages 574-579, Scindirect, 2011.
- [61] Harpreet Singh, Madan M. Gupta, Thomas Meitzler, Zeng-Guang Hou, Kum Kum Garg, Ashu M. G. Solo, and Lotfi A. Zadeh, “Real-Life Applications of Fuzzy Logic”, Journal of Advances in Fuzzy Systems, Article ID 581879, 2013. DOI: <http://dx.doi.org/10.1155/2013/581879>
- [62] Elmer P. Dadios, “Fuzzy Logic – Controls, Concepts, Theories and Applications”, ISBN: 978-953-51-0396-7, 2012. DOI: 10.5772/2662
- [63] **Nguyen Van Dong Hai**, Nguyen Thien Van, Nguyen Minh Tam, “Application of Fuzzy and PID Algorithm in Gantry Crane Control”, Journal of Technical Education Science, ISSN: 1859-127, No. 44A, pp. 48-53, November, 2017.

- [64] Sandeep D. Hanwate, Yogesh V. Hote, “Design of PID controller for IP using stability boundary locus”, Annual Indian Conference (INDICON), IEEE, Indian, 2014.
- [65] Mahadi Hasan, Chanchai Saha, Md. Mostafizur, Md. Rabiul Islam Sarker and Subrata K. Aditya, “Balancing of an IP using PD Controller”, Journal of Science, Dhaka University, 60(1), pp. 115-120, 2012.
- [66] C. Sravan Bharadwaj, T. Sudhakar Babu, N. Rajasekar, “Tuning PID Controller for IP using Genetic Algorithm”, Advances in Systems, Control and Automation, Part of the Lecture Notes in Electrical Engineering, pp. 395-404, Springer, Dec-2017.
- [67] Vishwa Nath, R. Mitra, “Swing-up and Control of Rotary IP using pole-placement with integrator”, Recent Advances in Engineering and Computational Sciences (RAECS), 2014.
- [68] Yan Lan, Minrui Fei, “Design of state-feedback controller by pole placement for a couple set of IPs”, 10th International Conference on Electronics Measurement & Instruments (ICEMI), pp. 69-73, 2011.
- [69] L. A. Zadeh, “Fuzzy sets”, Journal of Information and Control, Vol. 8, Issues 3, pp. 338353, Elsevier, 1965.
- [70] Sarah Spurgeon, “SMC: a tutorial”, European Control Conference (ECC), pp. 2272-2277, IEEE, France, June-2104.
- [71] **Nguyen Van Dong Hai**, Nguyen Minh Tam, Mircea Ivanescu, “A Method of Sliding Mode Control of Cart and Pole system”, Journal of Science and Technology Development, ISSN: 1859-0128, Vol. 18, Nr. 6, pp. 167-173, Vietnam, 2015.
- [72] Mircea Ivanescu, **Nguyen Van Dong Hai**, Nirvana Popescu, “Control algorithm for a class of systems described by TS-fuzzy uncertain models”, 20th International Conference on System Theory, Control and Computing (ICSTCC), 2016. (ISI proceeding)
DOI:10.1109/ICSTCC.2016.7790653
- [73] Sanket Kailas Gorade, Prasanna S. Gandhi, Shailaja R. Kurode, “Modeling and Output Feedback Control of Flexible IP on Cart”, International Conference in Power and Advanced Control Engineering (ICPACE), pp. 436-440, IEEE, 2015.
- [74] Chao Xu, Xin Yu, “Mathematical modeling of Elastic Inverted Pendulum control system”, Journal of Control Theory and Applications, Vol. 2, Issue 3, pp. 281-282, Springer, 2004.
- [75] Tang Jiali, Ren Gexue, “Modeling and Simulation of a Flexible IP System”, Journal of Tsinghua Science and Technology, vol. 14, Nr. 82, pp. 22-26, 2009.
- [76] Massachusetts Institute of Technology: “Laboratory Module. No. 1: Elastic behavior in Tension, bending, buckling, and vibration”, Spring, 2004.
- [77] **Nguyen Van Dong Hai**, Mircea Ivanescu, Mircea Nitulescu, “Observer-based Controller for Balancing Robot with Uncertain Model”, 17th International Carpathian Control Conference (ICCC), pp226-231, IEEE, May-2016. (ISI proceeding)

- [78] **Nguyen Van Dong Hai**, Mircea Ivanescu, Mihaela Florescu, Mircea Nitulescu, “Frequency criterion for balancing robot control described by uncertain models”, 20th International Conference on System Theory, Control and Computing (ICSTCC), pp. 134-137, IEEE, October-2016. (ISI proceeding)
- [79] Mircea Ivanescu, **Nguyen Van Dong Hai**, Nirvana Popescu, “Control Algorithm for a Class of Systems Described by T-S Fuzzy Uncertain Models”, pp. 129-133, IEEE, 2016. (ISI proceeding)
- [80] Yasemin Ozkan Aydin, “Optimal Control of a Half Circular Compliant Legged Monopod”, Doctor thesis. Middle East Technical University, 2013.
- [81] Nguyen Xuan Vu Trien, Le Thi Thanh Hoang, Nguyen Minh Tam, **Nguyen Van Dong Hai**, “Feedback Control Design for a Walking Athlete Robot”, Journal of Robotica & Management, ISSN: 1453-2069, Vol. 22, Nr. 1, June, 2017.
- [82] Ege Sayginer, “Modelling the Effects of Half Circular Compliant Legs on the Kinematics and Dynamics of a Legged Robot”, Master Thesis, Middle East University, 2010.
- [83] R. M. Murray, “CDS 110b, Lecture 2- LQR”, California Institute of Technology, 2006. Link: <https://www.cds.caltech.edu/~murray/courses/cds110/wi06/lqr.pdf>
- [84] **Nguyen Van Dong Hai**, Mircea Ivanescu, Mircea Nitulescu, “Controller based on Lyapunov for a Class of Running Robot”, 18th International Conference on Carpathian Control Conference (ICCC), pp. 107-111, July-2017.
- [85] Christian Paulsson, “Dasher the running robot”, Master thesis in electronics, control theory, 30p D-level, Malardalen University. Link: <http://www.idt.mdh.se/utbildning/exjobb/files/TR1021.pdf>
- [86] Link: <https://spectrum.ieee.org/automaton/robotics/humanoids/athlete-robot-learning-to-run-like-human>
- [87] Link: <http://www.control.toronto.edu/people/profs/bortoff/acrobot.html>
- [88] Link: http://www.idc-online.com/technical_references/pdfs/mechanical_engineering/Energy_Methods.pdf
- [89] Link: <https://eis.hu.edu.jo/ACUploads/10526/CH%204.pdf>
- [90] Scott C. Brown, Kevin M. Passino, “Intelligent Control of an Acrobot”, Journal of Intelligent and Robotic Systems, Vol. 18, Issue 3, pp. 209-248, 1997.
- [91] Jiri Zikmund, Sergej Celikovskiy, Claude H. Moog, “Nonlinear Control Design for the Acrobot”, IFAC Proceedings Volumes, Volume 40, Issue 20, pp. 446-451, 2007.
- [92] Ancai Zhang, Jinhua She, Xuzhi Lai, Min Wu, “Motion planning and tracking control for an acrobot based on a rewinding approach”, Journal Automatica (Journal of IFAC), Vol. 49, Issue 1, pp. 278-284, 2013.
- [93] Ikuo Mizuuchi, Yuto Nakanishi, Yoshinao Sodeyama, Yuta Namiki, Tamaki Nishino, Naoya Muramatsu, Junichi Urata, Kazuo Hongo, Tomoaki Yoshikai, and Masayuki Inaba,

- “An advanced musculoskeletal humanoid kojiro”, In *Proc. 7th IEEE-RAS Int. Conf. on Humanoid Robots (Humanoids 2007)*, pp 294–299, November 2007.
- [94] R.Niiyama, S.Nishikava, Y.Kuniyoshi, “Athlete Robot with Applied Human Muscle Activation Pattern for Bipedal Running Robot”, In *Proc. IEEE/RSJ Int.Conf. on Intelligent Robots and Systems (IROS 2009)*, pp 1092–1099, October 10–15, 2009.
- [95] Y.Gangamwar, V.Deo, S.Chate, M.Bahandare, H.Desphande, “Determination of Curved Beam Deflection by Using Castigliano’s Theorem”, *Int Journal for Research in Emerging Science and Technology*, Vol 3, Issue 5, , pp 19-24, 2016.
- [96] R.Cardozzo, M.Mezza, “Comparison of Two Fuzzy Skyhook Control Strategies Applied to an Active Suspension”, *Int. Journal of Computer Science and Software Engineering*, Vol 5, Issue 6,pp 108-113, 2016
- [97] H.Vasudevan, A.Dollar, J.Morell, Design for Control of Wheeled Inverted Pendulum Platforms, *Journal of Mechanisms and Robotics*, ASME 2015, Vol. 7 / 041005-1
- [98] Khalil, N. H., 2002, *Nonlinear Systems*, Prentice Hall, Upper Saddle River, NJ.
- [99] Marc H. Raibert., “*Legged Robots That Balance*”, The MIT Press, 1986.
- [100] Ryosuke Tajima, Daisaku Honda, and Keisuke Suga, “Fast running experiments involving a humanoid robot”, In *Proc. IEEE Int. Conf. On Robotics and Automation (ICRA2009)*, pp 1571–1576, May 2009.
- [101] Toru Takenaka, Takashi Matsumoto, TakahideYoshiike, and Shinya Shirokura. “Real time motion generation and control for biped robot —2nd report: Running gait pattern generation”, In *Proc. IEEE/RSJ Int. Conf. on Intelligent Robots and Systems (IROS 2009)*, pp 1092–1099, October 10–15, 2009.
- [102] R. McNeill Alexander and H. C. Bennet-Clark, “Storage of elastic strain energy in muscle and other tissues”, *Nature*, 265(5590):114–117, Jan 1977.
- [103] Sang-Ho Hyon, “Compliant terrain adaptation for biped humanoids without measuring ground surface and contact forces”, *IEEE Transactions on Robotics*, 25(1):171–178, Feb. 2009.
- [104] Hong Jie-Ren, “Balance Control of a Car-Pole Inverted Pendulum System”, Master theiss, National University of ChengKung, Taiwan, 2002.
- [105] Ashwani Kharola and Pavin Patil, “Fuzzy Hybrid Control of Flexible Inverted Pendulum (FIP) System using Soft-computing Techniques”, *Pertanika Journal of Science and Technology*, 25(4), pp. 1189-1202, 2017.
- [106] Yawei Peng, Jinkun Liu, Wei He, “Boundary Control for a Flexible Inverted Pendulum System based on a PDE Model”, *Asian Journal of Control*, Vol. 20, Issue 1, 2016.
- [107] Altendorfer, R., Moore, N., Komsuoglu, H. et al, “RHex: A Biologically Inspired Hexapod Runner” *Autonomous Robots* (2001) 11: 207.

LIST OF PUBLICATIONS

International Journal

1. Huynh Xuan Dung, Huynh Duong Khanh Linh, Vu Dinh Dat, Nguyen Thanh Phuong, Nguyen Minh Tam, **Nguyen Van Dong Hai**, “Application of Fuzzy Algorithm in Optimizing Hierarchical Sliding Mode Control for Pendubot System”, International Journal of Robotica & Management, ISSN-L: 1453-2069; Print ISSN: 1453-2069; Online ISSN: 2359-9855 ,Vol. 22, Nr. 2, Dec-2017.
2. Nguyen Minh Hoang, Ngo Van Thuyen, Nguyen Minh Tam, Le Thi Thanh Hoang, **Nguyen Van Dong Hai**, “Desiging Linear Feedback Controller for Elastic IP with Tip Mass”, International Journal of Robotica & Management, ISSN-L: 1453-2069; Print ISSN: 1453-2069; Online ISSN: 2359-9855 , pp. 27-32, Vol. 21, Nr. 2, December-2016.
3. **Nguyen Van Dong Hai**, Huynh Xuan Dung, Nguyen Minh Tam, Cristian Vladu, Mircea Ivanescu, “Hierarchical Sliding Mode Algorithm for Athlete Robot Walking”, Journal of Robotics, ISSN: 1687-9600 (Print), ISSN: 1687-9619 (Online), Article ID 6348980, Hindawi, December-2017. DOI: doi.org/10.1155/2017/6348980 (ISI/ESCI/SCOPUS journal).
Link: <https://www.hindawi.com/journals/jr/2017/6348980/>
4. Nguyen Xuan Vu Trien, Le Thi Thanh Hoang, Nguyen Minh Tam, **Nguyen Van Dong Hai**, “Feedback Control Design for a Walking Athlete Robot”, Journal of Robotica & Management, ISSN-L: 1453-2069; Print ISSN: 1453-2069; Online ISSN: 2359-9855, Vol. 22, Nr. 1, June, 2017.
5. Mihaela Florescu, **Van Dong Hai Nguyen**, Mircea Ivanescu, “Output Track Controller with Gravitational for a Class of Hyper-Redundant Robot Arms”, Journal of Studies in Informatics and Control, Romania, 2015 (ISI/SCIE journal)
Link: <https://sic.ici.ro/output-track-controller-with-gravitational-compensation-for-a-class-of-hyper-redundant-robot-arms/>

International Conference

1. **Nguyen Van Dong Hai**, Mircea Ivanescu, Mircea Nitulescu, “Hierarchical Sliding Mode Control for Balancing Athlete Robot”, 21st International Conference on System Theory, Control and Computing (ICSTCC 2017), Sinaia, Romania, Nov-2017.
2. **Nguyen Van Dong Hai**, Nguyen Minh Tam, Mircea Ivanescu, “Application in Genetic Algorithm in Identifying System Parameters for IP”, International Symposium of Electrical and Electronics Engineering, Ho Chi Minh city University of Technology, Vietnam October-2015.
3. Mircea Ivanescu, **Nguyen Van Dong Hai**, Nirvana Popescu, “Control algorithm for a class of systems described by TS-fuzzy unvertain models”, 20th International Conference on System Theory, Control and Computing (ICSTCC), 2016. (ISI proceeding). DOI:10.1109/ICSTCC.2016.7790653

4. M. Nitulescu, M. Ivanescu, S. Manoiu-Olaru, **Nguyen V. D. H.**, *Experiment Platform for Hexapod Locomotion*, Book of Mechanisms and Machine Science, Vol. 46, Part VIII: Robotics-Mobile Robots, pp. 241-249, Springer, 2017. DOI: 10.1007/978-3-319-45450-4.
5. M. Ivanescu, M. Nitulescu, **Nguyen V. D. H.**, M. Florescu, *Dynamic Control for a Class of Continuum Robotics Arms*, Book of Mechanisms and Machine Science, Vol. 46, Part XI: Robotics-Robotic Control System, pp. 361-370, Springer, 2017. DOI: 10.1007/978-3-319-45450-4.
6. **Nguyen Van Dong Hai**, Mircea Ivanescu, Mircea Nitulescu, “Observer-based Controller for Balancing Robot with Uncertain Model”, 17th International Carpathian Control Conference (ICCC), pp226-231, IEEE, May-2016. (ISI proceeding)
7. **Nguyen Van Dong Hai**, Mircea Ivanescu, Mihaela Florescu, Mircea Nitulescu, “Frequency criterion for balancing robot control described by uncertain models”, 20th International Conference on System Theory, Control and Computing (ICSTCC), pp. 134-137, IEEE, October-2016. (ISI proceeding)
8. Mircea Ivanescu, **Nguyen Van Dong Hai**, Nirvana Popescu, “Control Algorithm for a Class of Systems Described by T-S Fuzzy Uncertain Models”, pp. 129-133, IEEE, 2016. (ISI proceeding)
9. **Nguyen Van Dong Hai**, Mircea Ivanescu, Mircea Nitulescu, “Controller based on Lyapunov for a Class of Running Robot”, 18th International Conference on Carpathian Control Conference (ICCC), pp. 107-111, July-2017.

Vietnamese domestic paper

1. **Nguyen Van Dong Hai**, Nguyen Phong Luu, Nguyen Minh Tam, Hoang Ngoc Van, “Optimal Control for Quadruped IP”, pp. 18-23, Vol. 34, Journal of Technical Education Science, Vietnam, ISSN: 1859-1272, 2016.
2. Tran Hoang Chinh, Nguyen Minh Tam, **Nguyen Van Dong Hai**, “A Method of PID-FUZZY control for pendubot”, Journal of Technical Education Science, No. 44A, pp. 61-67, ISSN: 1859-127, November-2017.
3. Nguyen Minh Tam, **Nguyen Van Dong Hai**, Nguyen Phong Luu, Le Van Tuan, “Modelling and Optimal Control for Two-wheeled Self-Balancing Robot”, Journal of Technical Education Science, Vietnam, ISSN: 1859-1272, Vol. 37, pp. 35-41, 2016.
4. Ho Trong Nguyen, Nguyen Minh Tam, **Nguyen Van Dong Hai**, “Application of Genetic Algorithm in Optimization Controller for Cart and Pole System”, Journal of Technical Education Science, ISSN: 1859-127, No. 44A, pp. 41-47, November, 2017.
5. Vu Duc Ha, Huynh Xuan Dung, Nguyen Minh Tam, **Nguyen Van Dong Hai**, “Hierarchical Fuzzy SMC for a Class of SIMO Under-actuated Systems”, Journal of Technical Education Science, ISSN: 1859-1272, Vietnam, 2017.
6. Vu Dinh Dat, Huynh Xuan Dung, Phan Van Kiem, Nguyen Minh Tam, **Nguyen Van Dong Hai**, “A method of Fuzzy-SMC for Pendubot model”, Journal of Science and

Technology-University of Da Nang, Vietnam, ISSN: 1859-1591, No. 11 (120), Issue 1, pp. 12-16, 2017.

7. **Nguyen Van Dong Hai**, Nguyen Thien Van, Nguyen Minh Tam, “Application of Fuzzy and PID Algorithm in Gantry Crane Control”, Journal of Technical Education Science, ISSN: 1859-127, No. 44A, pp. 48-53, November, 2017.
8. **Nguyen Van Dong Hai**, Nguyen Minh Tam, Mircea Ivanescu, “A Method of Sliding Mode Control of Cart and Pole system”, Journal of Science and Technology Development, ISSN: 1859-0128, Vol. 18, Nr. 6, pp. 167-173, Vietnam, 2015.
9. Vo Anh Khoa, Nguyen Minh Tam, Le Thi Thanh Hoang, Nguyen Thien Van, **Nguyen Van Dong Hai**, “Model and Control Algorithm Construction for Rotary Inverted Pendulum in Laboratory”, Journal of Technical Education Science, ISSN: 18959-1272, 2018. (in Vietnamese) (accepted)
10. Vo Anh Khoa, Nguyen Minh Tam, Le Thi Thanh Hoang, Nguyen Thien Van, Mircea Ivanescu, **Nguyen Van Dong Hai**, “PID controller in Step-motion Control for Bipedal Robot with Elastic Legs”, Journal of Technical Education Science, ISSN: 1859-127, 2018. (accepted)