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Bayesian Variable Selection and Coefficients Estimation in Tobit Quantile Regression Model

A Thesis Submitted for the Field: Cibernetics and Statistics

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Abstract

The aim of this thesis is to carry out Bayesian variable selection and coefficients estimation in Tobit quantile regression model via new proposed methods.

In Chapter 2, a new formulation for Bayesian Lasso QReg has been proposed by employing the scale mixture of uniform distributions formulation. Then, a fully Bayesian treatment that leads to a simple and efficient Gibbs sampling algorithm with tractable full conditional posterior distributions has been developed. The superiority of the proposed approach (Lasso U) has been shown on simulation study and real data. Some extensions to this approach are discussed in Tobit quantile regression.

In Chapter 3, a new Bayesian Lasso Tobit quantile regression method for variable selection and coefficients estimation assigning an independent scale-mixture of uniform (SMU) distributions for the regression coefficients has been proposed. Then, a simple and efficient (Markov Chain Monte Carlo) MCMC algorithm has been presented for Bayesian sampler. Simulation studies and a real data set were used to investigate the performance of the proposed method. Both simulated and real data examples show that the proposed method performs quite well compared to the other methods in a variety of scenarios.

In Chapter 4, a simple and efficient MCMC for composite Tobit quantile regression model based on a mixture of an exponential and a scaled normal distribution of the skewed Laplace distribution has been developed. Simulation studies show that the proposed method is effective in coefficient estimation with different distributions. Based on the simulation studies and real data analysis, we argue that it is necessary to combine quantile information based on estimators at different quantiles to achieve efficiency gain.

In Chapter 5, Bayesian Tobit quantile regression model, and Bayesian composite Tobit quantile model have been used to analyse the Iraqi banks' investments data in two ways. Firstly, coefficients estimation via thirty Tobit quantile levels. Secondly, variable selection through determining the relative importance for independent variables in our model, via thirty Tobit quantile levels. On the other hand, the Bayesian composite Tobit quantile model approach is used via a groups of six composite Tobit quantile levels, also in two ways. Firstly: for modelling the relationship between Iraqi banks' investments and nine independent variables. Secondly: variable selection via a groups of six composite Tobit quantile levels. A set of conclusions has been derived from theoretical and application viewpoints.

Keywords:

- Tobit Regression
- Quantile Regression
- Tobit Quantile Regression
- Coefficients Estimation
- Variable Selection
- Bayesian Approach
- Gibbs Sampler
- MCMC algorithm
- Simulation Study
- Real Data
- Prior Distribution

- **SMN** Distribution
- **SMU** Distribution
- Iraqi Banks
- Banking Investments

Rezumat

Scopul acestei teze este de a efectua selecţia Bayesiană a variabilelor şi estimarea coeficienților în modelul de regresie cuantilică Tobit prin intermediul unor noi metode propuse. În capitolul 2, a fost propusă o nouă formulare pentru regresia cuantilică (Qreg) Bayesiană prin utilizarea unei formulări bazat pe o mixtură scalată de distribuţii uniforme. Apoi, a fost dezvoltată o abordare Bayesiană completă care conduce la un algoritm simplu și eficient de eșantionare Gibbs cu distribuții aposteriori condiționate uşor de manevrat. Superioritatea abordării propuse (Lasso U) a fost demonstrată prin simulare și pe date reale. Unele extensii la această abordare sunt discutate în regresia cuantilică Tobit.

În capitolul 3, a fost propusă o nouă metodă de regresie cuantilică Tobit Lasso Bayesiană pentru selecția variabilelor și estimarea coeficienților care asignează o mixtură scalată independentă de distribuții uniforme (SMU) pentru coeficienții de regresie. Apoi, a fost prezentat un algoritm MCMC (Markov Chain Monte Carlo) simplu și eficient pentru eşantionatorul Bayesian. Atât studii bazate pe simulare cât și un set real de date au fost utilizate pentru a investiga performanța metodei propuse. Ambele arată că metoda propusă se comportă destul de bine în comparație cu celelalte metode într-o varietate de scenarii.

În capitolul 4, a fost elaborat un MCMC simplu și eficient pentru modelul de regresie cuantilică Tobit compozit, bazat pe o mixtură între o distribuție exponenţială şi una normală scalată a distribuției Laplace asimetrice. Studiile de simulare arată că metoda propusă este eficientă în estimarea coeficienților cu distribuții diferite. Pe baza studiilor de simulare și a analizei datelor reale, susținem că este necesar să combinăm informațiile bazate pe estimatori la diferitele cuantilice pentru a obține un câștig de eficiență.

În capitolul 5, modelul Bayresian Tobit de regresie cuantilică și modelul Bayesian Tobit cuantilic compozit au fost folosite pentru a analiza datele privind investițiile băncilor irakiene, în două moduri. În primul rând, estimarea coeficienților prin treizeci de nivele cuantile Tobit. În al doilea rând, selectarea variabilelor prin determinarea importanţei relative a variabilelor independente din modelul nostru, prin intermediul a treizeci de nivele cuantile Tobit. Pe de altă parte, abordarea bazată pe modelul Bayesian compozit cuantilic Tobit este utilizată prin intermediul a șase grupuri de nivele cuantilice compozite Tobit, de asemenea în două moduri. În primul rând: pentru modelarea relației dintre investițiile băncilor irakiene și nouă variabile independente. În al doilea rând: pentru selecția variabilelor prin un grup de șase nivele cuantilice compozite Tobit. Un set de concluzii a fost dedus din punct de vedere teoretic și aplicativ.

Cuvinte cheie:

- regresie Tobit
- regresie cuantilă
- regresie Tobit cuantilică
- Estimarea coeficienților
- Selecția variabilelor
- Abordare Bayesiană
- Eșantionare Gibbs

- Algoritmul MCMC
- Studii de simulare
- Date reale
- Distributii a priori
- Distributii **SMN**
- Distributii **SMU**
- Bănci irakiene
- Investiții bancare

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1. Introduction

Modelling the relationship between the averages of a response variable (dependent variable) Y with the set of covariates X is not always convenient. In many application studies mean regression may be not appropriate to describe the behaviour of response variable (outcome variable) Y with the covariates X. For example, the effect of demographic properties and maternal conduct on the weight of infant born was studied by (Abrevaya, J., & Dahl, C. M. (2008))[6] in the United States. This study was focused on low birth weight for infants, which causes many health problems. This data was analysed by standard mean regression; the conditional mean was not an attractive approach for low tail distribution. Quantile regression (Q Reg) was proposed by (Koenker and Bassett (1978))[38] as an extension for standard mean regression in different conditional quantiles of a dependent variable.
Quantile regression model is capable of providing complete information about different quantiles of the stochastic relationships between dependent and predictors variables. Recently, Q Reg model has received much interest in theoretical and application studies. Q Reg model is applied in different fields such as: Microarray study (Wang and He, (2007))[69], agricultural economics (Kostov and Davidova, 2013)[41], ecological studies (Cade and Noon, (2003))[14], body mass index (Yu et al., 2013))[74], growth chart (Wei et al., (2006))[68], and so on "Fadel Hamid Hadi Alhusseini, 2017"[24]. The quantile regression models have good properties compared with other regression models. The Q Reg model belongs to a robust models family (Koenker and Geling, (2001))[43]. Quantile regression model does not require any assumption about the residual distribution providing greater statistical efficiency than other regression models when the error is non-normal "Fadel Hamid Hadi Alhusseini, 2017"[24]. Also, Q Reg model is robust against the economic problems. All these features made the Q Reg model of an important model in various application fields. The following mathematical formula belongs to Q Reg model.

$$y_i = x_i^T \beta_\theta + \varepsilon_i, \qquad \theta \in (0,1), \qquad [1\,]$$

For any θth quantile, $(0 < \theta < 1)$, the θth quantile regression can be denoted as $Q_{y_i|x_i}(\theta) = x'_i \beta_\theta$, where y_i is the response (dependent) variable, x_i^T is a k-dimensional vector of covariates (independent variables), β_θ is a coefficients vector of Q Reg model.

There is an infinite number of points which belong to $(0 < \theta < 1)$. So, there are infinite quantile levels; at each quantile levels the Q Reg model has been estimated. Therefore, the Q Reg model has a high flexibility and capability of providing a perfect information about the relationships between response variable and predictor variables at different quantile levels, unlike classical regression model in which only one regression line is estimated by conditional mean of the response variable (y) given x, $E\,(y|x)$. (Koenker and Hillock, 2001))[40]. This is shown in the two following figures:

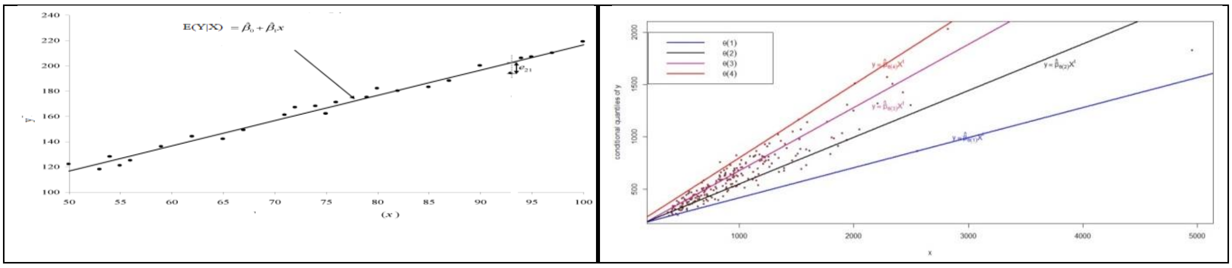

Figure 1: Regression lines which are estimated via classical regression model and Q Reg model.

The left figure shows the estimated regression line by classical regression model. Sometimes, the classical regression model cannot give us complete information about the relationship between the dependent variable and covariates (independent variables). The right figure shows four quantile regression lines estimated at four quantile levels. Always, the Q Reg model give us an obvious picture about the relationship between a response variable and covariates (explanatory variables), because many quantile regression lines are estimated. Each line belongs to a specific quantile level. The discussed matter focuses on the properties of Q Reg model. Another important matter in building the Q Reg model is the selection of active covariates. In recent years, the selection of important subsets of covariates has taken a lot of attention in the literature. Many methods of variable selection have been proposed, such as Lasso (Tibshirani, (1996))[64], SCAD (Fan and Li, (2001))[20], the elastic net method (Zou and Hastie, (2005))[75], adaptive Lasso (Zou, (2006))[]. Variables selection techniques are used also in Q Reg model, such as Lasso penalty, which was applied to the mixed-effect Q Reg model or longitudinal data by (Koenker (2004))[39], a solution path was introduced by (Li and Zhu (2008))[53] to L_1-penalized quantile regression model. To estimate the coefficients of Lasso Q Reg model (Li and Zhu (2008))[53] proposed the following equation.

$$\min_{\beta_\theta} \sum_{i=1}^{n} \rho_\theta(y_i - x_i^T \beta_\theta) + \lambda \parallel \beta_\theta \parallel, \qquad [2]$$

where $\rho_\theta(s)$ is the check(loss) function defined by $\rho_\theta(s) = s\{\theta - \mathrm{I}(s \le 0)\}, and$ where $I(s < 0)$ is the indicator function and $\lambda($ $(\lambda \ge 0)$ is the shrinkage parameter. Unfortunately, the equation (2) is not differentiable at zero, hence there is no exact solution for equation (2). (Koenker, (2005))[42] shows the minimization of (2) can be achieved by a linear programming algorithm (Koenker and D'Orey, (1987))[46].

Also to estimate the coefficients of Lasso Q Reg model the Bayesian approach is used. (Park and Casella (2008))[60] proposed the Bayesian Lasso in classical linear regression models via using a scale mixture of normal (SMN) prior distributions on the regression coefficients and independent exponential prior distributions on their variances "Fadel Hamid Hadi Alhusseini,2017"[24]. (Li et al, (2010))[52] suggested Bayesian Lasso Q Reg, also by using (SMN) prior distributions on the regression coefficients and independent exponential prior distributions on their variances. (Li et al, (2010))[52] proposed an efficient and simple Markov chain Monte Carlo (MCMC) algorithm for updating all model coefficients from posterior distribution. The Bayesian formulation is a flexible procedure of estimating the penalty parameter along with regression coefficients. Recently, for linear regression, (Mallick and Yi (2014))[55] provided a different process of Lasso-based model by using the scale mixture of uniform (SMU) as formulation of the Laplace density function. (Mallick and Yi (2014))[55] provided a new method to coefficients estimation and variable selection in classical regression model. This method performed quite well compared to some other existing methods in the same field.

Our contribution consists in a new formulation for Bayesian Lasso Q Reg by employing SMU formulation as a new prior distribution to coefficients of Q Reg model. We also developed a full Bayesian treatment which led to an efficient and simple Gibbs sampling algorithm with tractable full conditional posterior distributions. The full conditional posterior distributions of our Gibbs sampling algorithm were collected in two steps. Firstly the likelihood function which belongs to the asymmetric Laplace distribution (ALD) family, see (Yu and Moyeed (2001))[]. Here, we cannot use ALD directly because it would lead to hard accounts and inefficient Gibbs sampling algorithm. Therefore, we used alternative formula of ALD, proposed by (Kozumi and Kobayashi, (2011))[49] which is a scale mixture of normal distributions (SMN) "Fadel Hamid Hadi Alhusseini"[24]. The response variable (y_i) is distributed through normal distribution with mean ($x_i^T\beta_\theta + (1-2\theta)m_i$) and variance ($2\sigma^{-1}m_i$) as $\sim N(x_i^T\beta_\theta + (1-2\theta)m_i, 2\sigma^{-1}m_i)$:

$$f(y|y_i, x_i\beta, \sigma) = \frac{1}{\sqrt{4\pi\sigma^{-1}m_i}} Exp \frac{-(y_i - x_i^t\beta - (1-2\theta)m_i)^2}{4\sigma^{-1}m_i} \quad [3]$$

where m_i is exponential distribution with scale parameter $\theta(1-\theta)\sigma$. The majority of researchers in field of Bayesian quantile regression and Bayesian regularized quantile regression use the Kozumi and Kobayashi formulation. It provides a simple and efficient MCMC algorithm.

It is our contribution to put SMU prior distributions on β_j and exponential prior distributions assigned to σ^2 , assuming they are independent. We developed an alternative hierarchical Bayesian Lasso Q Reg. According to the work of (Mallick and Yi (2014))[55], the Laplace prior distribution to β_j can be given as

$$\frac{\lambda}{2} e^{\{-\lambda|\beta_j|\}} = \frac{\lambda}{2} \int_{u_j > |\beta_j|}^{\cdot} \lambda \; exp\{-\lambda u_j\}\, du_j \quad ,$$

$$= \int_{-u_j<\beta_j<u_j}^{,} \frac{1}{2u_j} \frac{\lambda^2}{\Gamma(2)} u_j^{2-1} \; exp\{-\lambda u_j\}\, du_j \quad , \quad [4]$$

$$= \int_{-u_j<\beta_j<u_j}^{,} \frac{\lambda^2}{2} \; exp\{-\lambda u_j\}\, du_j$$

where $\Gamma(2) = (2-1)! = 1$

The equation (4) considers reformulation of Laplace prior distribution to SMU. This procedure provide us comfortable account for our Gibbs sampling and the attractive MCMC algorithm. β_j is uniform distribution with parameters $(-u_j, u_j)$ and u_j is Gamma distribution with shape parameter (2) and rate parameter (λ); also the parameter λ is Gamma distribution with shape parameter (c) and rate parameter (d) "Fadel Hamid Hadi Alhusseini"[24]. From the information above, our Bayesian hierarchical model can be formulated as follows:

$$\begin{aligned} & m_i|\sigma \sim exp\{\theta(1-\theta)\sigma\}, \\ & \beta_j|u_j \sim Uniform\,(-u_j, u_j), \\ & u_j|\lambda \sim Gamma(2,\lambda)\,, \\ & \sigma \sim \sigma^{a-1} exp(-b\lambda) \\ & \lambda \sim \lambda^{c-1} exp(-d\lambda)\,. \end{aligned} \quad [5]$$

Here, $Exp(\theta(1-\theta)\sigma)$ refers to the exponential distribution with rate parameter $\theta(1-\theta)\sigma$.

Also σ is Gamma distribution with shape parameter (a) and rate parameter (b). Where, a,b,c and d are four fixed hyper parameters. From the likelihood function shown in equation (3) and hierarchical model shown in equation (5), we will obtain conditional posterior distribution of $\beta, m, (m_1, \dots m_n)^T, u = (u_1, \dots u_n)^T$ and λ can be updated using an efficient MCMC-based computation technique.

The conditional posterior distribution of β is truncated normal, with mean $\left(\sigma_{\beta_j}^2 \sum_{i=1}^{n} \frac{\sigma x_{ij}\,\left(y_i-(1-2\theta)m_i-\sum_{,j\neq l}^{p} x_{ij}\beta_j\right)}{2m_i}\right) I\{|\beta_j| < u_j\}$ and variance $\left(\sum_{i=1}^{n} \frac{\sigma x_{ij}^2}{2m_i}\right)^{-1}$. The conditional posterior distribution of m is inverse Gaussian distribution with mean $1\Big/\sqrt{(y_i - x_i^t\beta)^2}$ and shape parameter $(\frac{\sigma}{2})$. Conditional posterior distribution of u_j is a left-truncated exponential distribution given by $u_j|\beta,\lambda \sim Exp(\lambda) I\{u_j > |\beta_j|\}$. Conditional posterior distribution of the penalty parameter λ is Gamma distribution with shape parameter $(c + 2p)$ and rate parameter $(d + \sum_{j=1}^{p} |\beta_j|)$. Also the conditional posterior distribution of σ is Gamma distribution with shape parameter $(a + \frac{3n}{2}\ )$ and rate parameter $(\sum_{i=1}^{n} \left(\frac{(y_i - x_i^t\beta + (1-2\theta)m_i)^2}{4m_i} + \theta(1-\theta)m_i\right) + b)$. From the full conditional posterior distributions we proceed to sample each unknown parameter $(\beta\,, m, (m_1, \dots m_n)^T, u = (u_1, \dots u_n)^T$, λ and σ. We will obtaine a good Gibbs sampler for coefficient estimations and variable selection in new Bayesian Lasso quantile regression model.

In order to evaluate the performance of our proposed method New Bayesian Lasso quantile regression (LassoU) it is compared with three other methods (LassoN, rq and MCMCquantreg) via the simulation approaches and real data, where the simulation approaches take the following description:

In first simulation, our simulation data are generated from sparse model:

$$y_i = 3x_{1i} + 1x_{2i} + 2x_{5i} + \varepsilon_i, \qquad [i = 1,2, \dots ., 100]$$

where y_i is the response variable, and our true parameters are $\beta = (0,3,1,0,0,2,0,0,0)$. The eight covariates $(x_{1i}, x_{2i}, x_{3i}, x_{4i}, x_{5i}, x_{6i}, x_{7i}, x_{8i})$ are simulated from a multivariate normal with mean 0 and $cov(x_h, x_g) = 0.5^{|h-g|}$. The random error distributions (ε_i) are generated from a $\chi^2_{(3)}$ distribution with three degrees of freedom, a $t_{(3)}$ Distribution with three degrees of freedom and normal distribution with $mean\ (\mu)$and variance (9), $\varepsilon_i \sim N(\mu, 9)$. The methods under testing are evaluated by the median of mean absolute deviations (MMAD), where MMAD $= median\left(mean\left(\left|x^T\hat{\beta} - x^T\beta^{true}\right|\right)\right)$, and the standard deviations (SDs). In first simulation study and under three choice of the error distribution, our algorithm was run for 11000 iterations and the first 1000 were removed. The results were as follows : the performance of our propose method New Bayesian Lasso Q Reg (LassoU) appears very good compared with Bayesian and non-Bayesian methods (LassoN, rq and MCMCquantreg). In general, the MMAD generated by New Bayesian Lasso Q Reg is much smaller than the MMAD generated by other three methods (LassoN, rq and MCMCquantreg), at all quantile levels and all distributions under consideration. Also, in the first simulation study with different error distributions, the SD obtained from our proposed method is much smaller than the SD obtained from other approaches (LassoN, rq and MCMCquantreg). Our MCMC algorithm is very stable and this is clear from the coefficients of multivariate potential scale reduction factor (MPSRF).Where, MPSRF is stable and close to 1 after about 2000 iterations. This shows that the convergence of the full conditional posterior distribution for the proposed method was quick and the mixing was good.

In the second simulation, our simulation data are generated from sparse model:

$$y_i = 3x_{1i} + \varepsilon_i, \qquad [i = 1,2, \dots ., 100]$$

where the true parameters are $\beta = (0,3,1,0,0,2,0,0,0)^T$. Like in first simulation, 8 covariates $(x_{1i}, x_{2i}, x_{3i}, x_{4i}, x_{5i}, x_{6i}, x_{7i}, x_{8i})$ are simulated from a multivariate normal with mean 0 and $cov(x_h, x_g) = 0.5^{|h-g|}$. The MMAD is computed by (LassoU) which is much smaller than MMAD which is computed by Bayesian and non-Bayesian methods (LassoN, rq and MCMCquantreg), via three quantile levels and three different error distributions. Also the results of the SD is computed by our proposed method (LassoU), which is smaller than MMAD computed by other three methods. From the results of the second simulation, we conclude our proposed method (LassoU) has a good performance compared with Bayesian and non-Bayesian (LassoN, rq and MCMCquantreg).

In the third simulation, our simulation data are generated from dense model:

$$y_i = x_{ij}^T \beta_\theta + \varepsilon_i, \qquad [i = 1,2, \dots, 100, j = 1, \dots, 8]$$

where $\beta = (0.00, 0.85, ,0.85, ,0.85, ,0.85, ,0.85, ,0.85, ,0.85, ,0.85)^T$. Like in first and second simulations, eight covariates $(x_{1i}, x_{2i}, x_{3i}, x_{4i}, x_{5i}, x_{6i}, x_{7i}, x_{8i})$ are simulated from a multivariate normal with mean 0 and $cov(x_h, x_g) = 0.5^{|h-g|}$. The MMAD and SD are computed by (LassoU) and are much smaller than MMAD that is computed by Bayesian and non-Bayesian methods (LassoN, rq and MCMCquantreg), via three quantile levels and three different error distributions. From the results of third simulation, we conclude that also our proposed method (LassoU) has a better performance compared with Bayesian and non-Bayesian (LassoN, rq and MCMCquantreg).

Also, from the three simulation results, our proposed method (LassoU) has smallest MMADs and SD compared with three other methods. Therefore, our proposed method has higher accuracy than the other methods in coefficients estimation and variable selection.

Also for evaluating our proposed method, we could use the estimation of β in direct way. We only choose the case of the random error, $\varepsilon_i \sim N(\mu, 9)$, at three different quantile levels ($\theta_1 = 0.50, \theta_2 = 0.75$, and $\theta_3 = 0.95$). The estimates of β by our proposed method, a new Bayesian Lasso Q Reg (LassoU) is very close to the true parameter values, compared with other methods such as LassoN, rq and MCMCquantreg. In general, LassoU performs well in estimating the regression coefficients compared with Bayesian and non-Bayesian methods (LassoN, rq and MCMCquantreg). Our simulation scenarios show that the Lasso U is effective in variable selection and coefficient estimation in quantile regression model. The simulations studies also indicate that our proposed method (Lasso U) is robust when the distribution which belongs to the error term is not ALD "Fadel Hamid Hadi Alhusseini ,2017"[24].

Our proposed method is assessed with real data. For this purpose, we used air Pollution Data which contains response variable in the log (concentration of NO2 per hour), and seven covariates - x_1 (log (number of cars per hour)), x_2 (temperature), x_3 (wind speed in meters per second), x_4 (the temperature difference), x_5 (wind direction), x_6 (time of day in hours) and x_7 day number. For evaluating the performance our LassoU method, we compared it with three other methods (LassoN, rq and MCMCquantreg).The mean square error (MSE) criterion has been used via three quantile levels $\theta \in \{0.50, 0.75, 0.95\}$. Generally, the (MSE) and SD computed by our proposed new Bayesian Lasso Q Reg were smaller than (MSE) computed by three other methods (LassoN, rq and MCMCquantreg) in majority of quantile levels. Therefore, our proposed method new Bayesian Lasso Q Reg has a better performance compared with Bayesian and non-Bayesian methods (LassoN, rq and MCMCquantreg). From both the simulation and real data scenarios, our proposed method a new Bayesian Lasso quantile Q Reg can be considered quite a new extension to coefficients estimation and variable selection in Q Reg model.

We also proposed a new contribution to coefficients estimation and variable selection in Tobit quantile regression (Tobit Q Reg) model. Since the Tobit regression model was proposed by (Tobin, (1958))[66], it became known in a variety of scientific fields (Greene, W (2010))[31], like in economic sciences, medical sciences, financial sciences, social sciences and engineering sciences. Tobit regression models offer limited information on the response variable. Therefore, at censored point equal to zero ($c = 0$), the Tobit regression model is defined as:

$$y_i = \begin{cases} T_i^* = \alpha + \beta x_i^T + \varepsilon_i & if\ T_i^* > 0 \\ 0 & if\ T_i^* \leq 0 \end{cases} \quad [6]$$

where $\varepsilon_i \sim N(0, \sigma^2)$, $[i = 1,2, \ldots\ldots, n]$

y_i is the censored response variable limited at the censored point equal to $0.$ T_i^* is the latent variable, x_i^T is $1 \times k$ a vector of the explanatory variables (covariates), α is the intercept term, β is the vector of unknown parameters in the Tobit regression model and ε_i is a random error term distributed according to the normal distribution with mean zero and variance (σ^2). Therefore, the latent variable T_i^* is distributed normally with the mean ($\alpha + \beta x_i$) and the variance(σ^2), where $T_i^* \sim N(\alpha + \beta x_i, \sigma^2)$. From the equation (6), the Tobit regression model is formed from two parts. First, when the latent variable T_i^* is observed, i.e., $T_i^* > 0$, the probability density function (pdf) of latent variable T_i^* belongs to observed (non-limited observation) latent variable, where $y_i = T_i^* \quad if\ T_i^* > 0$ and it is:

$$f(y_i) = \frac{1}{\sqrt{2\pi\sigma^2}} \; e^{\frac{-(y_i - \alpha + x_i^T \beta)^2}{2\sigma^2}} \quad [7]$$

The equation (1.7) can be reformulated as in equation (8)

$$f(y_i) = \frac{1}{\sigma} \phi \left(\frac{y_i - (\alpha + x_i^T \beta)}{\sigma} \right) \quad [8]$$

where $\phi(.)$ is a probability density function (pdf) which is belong to observed latent variable. The second part is dedicated to unobserved latent variable (T_i^*) when $T_i^* \leq 0$, So, it will take the cumulative distribution function of the normal distribution. $pro(y_i = 0) \quad if \quad pro(T_i^* \leq 0) \longrightarrow \Phi\left(\frac{y_i - (\alpha + x_i^T \beta)}{\sigma}\right) = \Phi\left(\frac{0 - (\alpha + x_i^T \beta)}{\sigma}\right)$

$$= \Phi\left(\frac{-(\alpha + x_i^T \beta)}{\sigma}\right) = 1 - \Phi\left(\frac{(\alpha + x_i^T \beta)}{\sigma}\right) \quad [9]$$

$\Phi(.)$ is a cumulative distribution function (cdf) (Greene,W(1999))[30].

The Tobit regression model is a mixed function between probability density function and cumulative distribution function of the normal distribution. Tobit regression model is a good statistical tool for modelling the relationship between censored response variables and a set of explanatory variables (covariates). But the Tobit regression model is very sensitive to regression problems. The Tobit regression model is not sufficient when the data contains outlier values. Also, Tobit regression model cannot provide good estimators when the normal assumptions are not achieved. To avoid these hard matters, Tobit quantile regression (Tobit Q Reg) model proposed by (Powell (1986))[59] has several properties. The Tobit Q Reg analyses the entire conditional distributional features of the dependent (response) variable. Tobit quantile regression model is focused on a set of quantile functions at different quantile levels. Therefore, it has flexibility and ability to give us a complete picture of the full distribution of the relationship between the response variable and explanatory variables. Tobit

Q Reg model is a normal extension to the classical Tobit regression model. Tobit Q Reg model has attractive properties which make it an important model for describing the relationship between censored response variable and a set of explanatory variables. The mathematical formulation of Tobit Q Reg model can be written as:

$$y_i = \begin{cases} T_i^* = \alpha_\theta + x_i^T \beta_\theta + \varepsilon_i & if\ T_i^* > 0 \\ 0 & if\ T_i^* \leq 0 \end{cases} \quad [10]$$

Where θ is $(0 < \theta < 1)$. The Tobit Q Reg model can have another mathematical formulation:

$$y = \max(C, T_i^*), \quad where\ T_i^* = \alpha_\theta + x_i^T \beta_\theta + \varepsilon_i \quad and\ C\ is\ equal\ zero\ .$$

where T_i^* is latent response variable, $(\alpha_\theta, \beta_\theta)$ are intercept and unknown parameters of the Tobit quantile regression respectively, $\theta \in (0,1)$. For coefficients estimation in Tobit Q Reg minimized the following loss function:

$$\underset{\alpha_\theta,\beta_\theta}{min} = \sum_{i=1}^{n} \rho_\theta \left(y_i - max\{0, T_i^*\}\right) \quad [11]$$

where $\rho_\theta(\varepsilon)$ is called check function of (Koenker and Bassett (1978))[38] at a quantile θ, the equation (11) also is not differentiable at 0. But minimization of equation [11] can be resolved by a linear programming algorithm (Koenker and D'Orey, (1987))[46] to give us coefficients estimation of Tobit Q Reg model. Although asymptotic properties for Tobit Q Reg are studied and many algorithms are proposed, the majority of these algorithms are inefficient, when the response variable has too much censored data. Presently, a possible estimation of coefficients of Tobit Q Reg model by (crq) function exists in the package (quanTobit Req) (Koenker, (2011))[45]. Recently, (Yu and Stander, (2007))[] have proposed a Bayesian approach to coefficients estimation of Tobit Q Reg, even when there is much censored Data. In recent years, selection of important subset of explanatory variables has taken a lot of attention in the literature for coefficients estimation with Tobit Q Reg Lasso penalty as follows:

$$\underset{\alpha_\theta,\beta_\theta}{min} = \sum_{i=1}^{n} \rho_\theta \left(y_i - max\{0, \alpha_\theta + x_i^T \beta_\theta + \varepsilon_i\}\right) + \lambda \parallel \beta_\theta \parallel \quad [12]$$

where (α_θ, β_θ) are intercept term and unknown parameters vector respectively, and $\lambda \parallel \beta_\theta \parallel$ is penalty for the estimation and selection of Tobit quantile coefficients. Also the minimization of equation [12] can be resolved by a linear programming algorithm, or by the Bayesian penalized model in Tobit Q Reg to achieve coefficients estimation and variable selection in Tobit Q Reg models. Bayesian adoptive Lasso model in Tobit quantile regression was developed by (Alhamzawi (2013))[4], which in (2014) proposed also the Bayesian adaptive elastic net Tobit Q Reg.

Our contribution consists in the fact that we proposed a new Bayesian Lasso Tobit Q Reg to achieve variable selection and coefficients estimation in Tobit quantile regression model by using Bayesian approach. Most methods in the field of penalized Bayesian Tobit Q Reg assigned scale mixture of normal distribution (SMN) prior to achieving Bayesian Lasso in Regression models. (Mallick and Yi, (2014))[55] provided a new technique for achieving Bayesian Lasso in traditional regression model by SMU prior, instead of the Laplace density function. We developed a new Bayesian Lasso in Tobit Q Reg model via using (SMU) prior distribution to Tobit Q Reg model coefficients. Our proposed method generated new conditional posterior distributions, which are very important for constructing efficient algorithm MCMC. For this proposed method we were inspired from the suggestion of

(Konker and Machado (1999))[48] and (Yu and Moyeed (2001))[]. These researchers observed the convergence between loss function (10) and skew-Laplace distribution (SLD) (asymmetric Laplace distribution). Therefore, the random error term ε_i is distributed as asymmetric Laplace distribution with probability density function (pdf), taking the following formula:

$$f(\varepsilon_i|\mu,\sigma,\theta) = \frac{\theta(1-\theta)}{\sigma}\ \exp{-\rho_\theta}\left\{\left(\frac{\varepsilon_i-\mu}{\sigma}\right)\right\} \qquad [13]$$

When the mean is equal 0, and the variance is equal to 1, then the probability density function which relates to random error ε_i is as follows:

$$f(\varepsilon_i|\sigma,\theta) = \theta(1-\theta)\ \exp{-\rho_\theta}\{(\varepsilon_i)\} \qquad [14]$$

$\rho_\theta(.)$ is the check (loss) function. The joint distribution of response variable $y = (y_1,\dots,y_n)^T,\ given\ X = (x_1,\dots,x_n)^T$, is:

$$f(y|X,\alpha,\beta,\sigma,\theta) = \theta^n(1-\theta)^n exp\left\{-\sum_{i=1}^{n}\rho\theta(y_i - max\{0,\alpha_\theta + x_i^T\beta_\theta + \varepsilon_i\}\right\} \qquad [15]$$

Maximizing the likelihood function of the equation [15] is equivalent to minimizing the equation [11]. By using ALD directly this leads to hard computations. Therefore, (Kozumi and Kobayashi, (2011))[49] suppose that the ALD can be reformulated in scale mixture normal distribution. The likelihood function of equation [14] becomes as follows:

$$f(T_i^*|\alpha_\theta,x_i^T,\theta,\beta_\theta,m_i) = \left[\frac{1}{\sqrt{4\pi m_i}}\right]^n e^{-\sum_1^n\frac{\left(T_i^*-\alpha_\theta-x_i^T\beta_\theta-(1-2\theta)m_i\right)^2}{4m_i}} \qquad [16]$$

where m_i is the experiential distribution density function with rate parameter $\theta(1-\theta)$. The equation [16] is an important part for building our Gibbs samplers. (Tibshirani, (2011))[65] introduces the Laplace prior distribution assigned to Bayesian Lasso framework. Using Laplace prior distribution directly leads to hard computation of full conditional posterior distributions. Therefore, using simplified formulas to Laplace prior distribution leads to simple and efficient MCMC algorithm. There is an alternative SMN formula proposed by (Andrews and Mallows, (1974))[5], as follows:

$$\frac{\lambda_j}{2}\ e^{-\lambda_j|\beta_j|} = \int_0^\infty \frac{1}{\sqrt{2\pi s_j}} e^{\left(-\frac{\beta_j^2}{2s_j}\right)}\ \frac{\lambda_j^2}{2} e^{\left(-\frac{s_j\lambda_j^2}{2}\right)} ds_j \qquad [17]$$

The equation (17) is a simple formula of Laplace prior distribution with two functions. The first function can be assigned to prior distribution for parameters(β_j), which take normal distribution with mean zero and variance (s_j) as follows:

$$p(\beta_j|s_j) = \frac{1}{\sqrt{2\pi s_j}} exp\left\{-\frac{\beta_j^2}{2s_j}\right\} \qquad [18]$$

where the unknown variance of β_j is s_j . The exponential prior distribution for s_j takes the following form:

$$p(s_j|\lambda_j)\ \propto\ \frac{\lambda_j}{2}\ exp\left\{-\frac{s_j\lambda_j}{2}\right\} \qquad [19]$$

In the end, the scale mixture of normal prior distribution is consider a good alternative for Laplace prior distribution. This give us a simple computations to full conditional posterior distributions. This is why most researchers used SMN prior distribution in Bayesian penalized regression models. For instance, (Park and Casella, (2008))[60] presented the Bayesian Lasso in traditional regression model. These methods were extended to Tobit quantile regression. For instance (Alhamzawi, (2013))[4] proposed adaptive Lasso in Tobit quantile regression by using the Bayesian technique. Also (Alhamzawi and Yu, (2014))[2], suggested a Bayesian technique for coefficient estimation in Tobit Q Reg model, by utilizing g-prior distribution with ridge parameter. Also, (Alhamzawi, (2014))[2] proposed a Bayesian elastic net penalty in Tobit Q Reg. In our proposed method, we used SMU prior distribution as alternative formula about Laplace prior distribution for coefficients in our model:

$$\frac{\lambda}{2} e^{\{-\lambda|\beta_j|\}} = \int_{s_j>|\beta_j|}^{\infty} \frac{1}{2u_j} \frac{\lambda^2}{\Gamma(2)} \; {u_j}^{2-1}\, exp\{-\lambda u_j\}\, du_j \;\; ,$$

$$= \int_{|\beta_j|}^{\infty} \frac{1}{2u_j} \frac{\lambda^2}{\Gamma(2)} \; {u_j}^{2-1}\, exp\{-\lambda u_j\}\, du_j \qquad [20]$$

The equation (20) has two functions. The first function is assigned to uniform prior distribution for u_j**,** and the second function is assigned to Gamma prior distribution with shape parameter (2) and scale parameter (λ). The scale parameter (λ) has Gamma prior distribution with parameters (a, b). This parameter is necessary for coefficients shrinkage to close from zero. The prior distribution of α_θ is assigned to standard uniform prior distribution. The parameters a, b, are fixed hyper parameters which take initial values.

Therefore, our Bayesian hierarchical approach with SMU for our model parameters will be as follows:

$$y_{i=max\{0,T_i^*\}}, \qquad i=1,\ldots,n,,$$

$$T_i^*|\alpha_\theta, \beta_\theta, z_i \sim N(\alpha_\theta + x_i^T\beta_\theta + (1-2\theta)m_i, 2m_i),$$

$$p(\alpha_\theta) \propto 1,$$

$$m_i \sim Exp\left(\theta(1-\theta)\right), \qquad [21]$$

$$\beta_j|u_j \sim Uniform\left(-u_j, u_j\right),$$

$$u_j|\lambda \sim {u_j}^{2-1}\exp(-\lambda u_j),$$

$$\lambda \sim Gamma(a, b),$$

The Bayesian hierarchy in equation (21) is an important part to generate our Gibbs sampler. The first part is dedicated to the likelihood function as in equation (21) and the second part will be shown in Bayesian hierarchy in equation (21) , where we will obtain our conditional posterior distributions which are output from mathematical formula as follows:

$$posterior \;\; distribution \; \propto likelihood\, function \; \times \; prior\, distribution$$

Under the hierarchy of Bayesian models (21) and (16), the Gibbs sampler algorithm is used to sample and update the parameters. The full conditional posterior distributions for our proposed method will be as follows:

The conditional posterior distribution of variable (T_i^*) follows the truncated normal distribution which is given by:

$$T_i^*|y_i, x_i, m_i, \alpha_\theta, \beta_\theta \sim \begin{cases} \gamma(y_i) & if\ y_i > o \\ N(\alpha_\theta + x_i^T\beta_\theta + (1-2\theta)m_i, 2m_i)I(T_i^* \le 0) & \text{otherwise} \end{cases} \quad [22]$$

where $\gamma(y_i)$ is degenerate distribution. The full conditional posterior distribution of α_θ is normal distribution with mean equal to $(\sum_{i=1}^{n} \frac{(T_i^* - x_i^T\beta_\theta - (1-2\theta)m_i)}{2m_i})$ and variance equal to $(\sum_{i=1}^{n}\left(\frac{1}{2m_i}\right))$. Also, the full conditional posterior distribution of m_i is inverse Gaussian with mean $\sqrt{\frac{\mathbf{1}}{(T_i^* - \alpha_\theta - x_i^T\beta_\theta)^2}}$ and shape parameter $(\frac{\mathbf{1}}{\mathbf{2}})$. And the full conditional posterior distribution of β_j is truncated normal with mean $\acute{\sigma}^2 \sum_1^n \frac{x_{ij}\left(T_i^* - (1-2\theta)m_i - \sum_{j=1, j\neq k}^{p} x_{ij}\beta_j\right)}{2m_i}$ $\mathrm{I}\left[\left|\beta_j\right| < u_j\right]$ and variance $\acute{\sigma}^2 = \left(\sum_1^n \frac{x_{ij}{}^2}{2z_i}\right)^{-1}$. The full conditional posterior distribution of u_j is left-truncated exponential with rate parameter(λ). The full conditional posterior distribution of λ is a Gamma distribution with rate parameter $(a + 2p)$ and scale parameter$\left(b + \sum_{j=1}^{p}\left|\beta_j\right|\right)$, where a,b are hyperparameters which take initial values. Our Bayesian hierarchical posteriors will generate an attractive MCMC algorithm for our proposed method new Bayesian Lasso Tobit quantile regression (New B L Tobit Q Reg) "Fadel Hamid Hadi Alhusseini", 2017)[25].

We tested the performance of our proposed method through comparing it with two other methods. They are classical Tobit Q Reg method (crq) by using the crq() function employing Powell's approach in(Koenker (2013))[50] and Bayesian adaptive elastic net Tobit Q Reg (BAnet) which is reported by (Alhamzawi (2014))[2]. For testing these methods under comparison, we will use simulation approach and real data. In simulation approaches, we will use two criterions. The first is the Root Mean Square Residual, $\text{RMSR}(\beta) = \sqrt{\frac{1}{S}\sum_{i=1}^{S}(\hat{\beta}_{kj} - \beta_j{}^{True})^2}$, $j = 1,2, \ldots\ldots p$. where S is the number of simulations, $\hat{\beta}_{kj}$ are the estimated parameters for j_{th} of model parameters in k_{th} of iterations and $\beta_j{}^{True}$ are true parameters (Lawrence and Arthur (1990))[54]. The second criterion we use is standard deviation of the MADS. The performance of our proposed method is investigated by simulation approaches. The true model used in these simulations is defined as follows:

$$y = \max(C, T_i^*)\ , \qquad i = 1,2, \ldots\ldots\ldots n\ , n = 100,$$

$$where\ T_i^* = \alpha_\theta + x_i^T\beta_\theta + \varepsilon_i,\ \ and\ C\ is\ equal\ zero\ .$$

θ is Tobit quantile level . In our simulations, we used three Tobit quantile levels are lower quantile level at $\theta_1 = 0.30$, middle quantile level at $\theta_2 = 0.60$ and higher quantile level at $\theta_3 = 0.90$. The error term ε_i , $i = 1,2 \ldots\ldots n$, is generated from three different distributions: a standard normal distribution, a $\chi_{(4)}^2$ distribution with four degrees of freedom and stander Laplace distribution $u_i \sim Laplace\ (0,1)$. The number of simulations was 100 for each case. Our algorithm was run 13000 iterations. The first 3000 were ruled out as it burnt in. For assessing the performance of our proposed method, it was compared with two other methods via four simulations approaches. In the first simulation approach, our simulation data was generated from very sparse case and with adding the intercept term to the true parameters $\beta = (0,5,0,0,0,0,0,0,1)^T$. The true model will be as follows:

$$y = \max(0, T_i^*)\ , \qquad T_i^* = 0 + x_{1i} + x_{8i} + \varepsilon_i, \qquad i = 1,2, \ldots\ldots\ldots 100$$

We simulated the explanatory variables $(x_{1i}, x_{2i}, x_{3i}, x_{4i}, x_{5i}, x_{6i}, x_{7i}, x_{8i})$ from a multivariate Gaussian distribution $X \sim N_8(\mu, \Sigma)$, where μ is the mean vector $\mu \epsilon R^n$ and Σ is covariance matrix with$(\Sigma_x)_{ij} = (2^{-1})^{|i-j|}$. Our proposed method (New B L Tobit Q Reg) has the smallest RMSR compared with BAnet and crq at different error distributions and different Tobit quantile levels. The proposed method has also smaller standard deviation (SD) for different error distributions and different quantile levels compared with other two methods. This means our proposed (New B L Tobit Q Reg) has a better performance and higher accuracy in coefficients estimation and variable selection in Tobit Q Reg model compared with other methods.

In the second simulation approach, our simulation data are generated from dense case and with add intercept term to true parameters$\beta = (0,0.85,0.85,0.85,0.85,0.85,0.85,0.85,0.85)^T$. Therefore, the true model is given as follows:

$y = \max(0, T_i^*)$, $T_i^* = 0 + 0.85x_{1i} + 0.85x_{1i} + 0.85x_{2i} + 0.85x_{3i} + 0.85x_{4i} + 0.85x_{5i} + 0.85x_{6i} + 0.85x_{7i} + 0.85x_{7i} + \varepsilon_i, \qquad i = 1,2, \ldots \ldots..100$

The explanatory variables $(x_{1i}, x_{2i}, x_{3i}, x_{4i}, x_{5i}, x_{6i}, x_{7i}, x_{8i})$ are simulated according to a multivariate Gaussian distribution $X \sim N_8(\mu, \Sigma)$, where μ is the mean vector $\mu \epsilon R^n$ and Σ is cov-matrix with $(\Sigma_x)_{ij} = (2^{-1})^{|i-j|}$. In the second simulation we can see that our proposed method (New B L Tobit Q Reg) has performance better than Bayesian and non- Bayesian methods (BAnet and crq, respectivelly), since the Root Mean Square Residual (RMSR) generated by our proposed method (New B L Tobit Q Reg) is very small compared with the other two methods at different error distributions. In the third simulation our data are simulated from group structures, including the intercept term, and $\beta = (0, (0,0,0), (2,2,2), (0,0,0), (2,2,2), (0,0,0))^T$. The following true model has been used.

$y = \max(0, T_i^*)$, $T_i^* = 0 + 2x_{4i} + 2x_{5i} + 2x_{6i} + 2x_{10i} + 2x_{11i} + 2x_{12i} + \varepsilon_i, \qquad i = 1,2, \ldots \ldots..100$

The explanatory variables $(x_{1i}, x_{2i}, x_{3i}, x_{4i}, x_{5i}, x_{6i}, x_{7i}, x_{8i,} x_{9i,} x_{10i,} x_{11i,} x_{12i,} x_{13i,} x_{14i,} x_{15i})$ are simulated from a multivariate Gaussian distribution $X \sim N_{15}(\mu, \Sigma)$,where μ is mean vector $\mu \epsilon R^n$ and Σ is covariance atrix with $(\Sigma_x)_{ij} = (2^{-1})^{|i-j|}$.

From the results which are listed from third simulation, we can see that the performance of our proposed method (New B L Tobit Q Reg) is better than Bayesian and non- Bayesian methods (BAnet and crq). This is clear from the resulted Root Mean Square Error (RMSR), where the RMSR computed by our proposed method is smaller than RMSR computed by the two other methods (BAnet and crq) for all quantile levels and all different error distributions.

Fourth simulation:

Our simulation data are generated from multi group structures, including the intercept term:

$\beta = (0, (0,0,0), (2,2,2), (0,0,0), (2,2,2), (0,0,0), (0,0,0), (2,2,2), (0,0,0), (2,2,2), (0,0,0))^T$.

Therefore, the true model will be as follows:

$y = \max(0, T_i^*)$, $T_i^* = 0 + 2x_{4i} + 2x_{5i} + 2x_{6i} + 2x_{10i} + 2x_{11i} + 2x_{12i} + 2x_{19} + 2x_{20i} + 2x_{21i} + 2x_{25} + 2x_{26i} + 2x_{27i} + \varepsilon_i, \qquad i = 1,2, \ldots \ldots..100$

where the explanatory variables

$(x_{1i}, x_{2i}, x_{3i}, x_{4i}, x_{5i}, x_{6i}, x_{7i}, x_{8i,} x_{9i,} x_{10i,} x_{11i,} x_{12i,} x_{13i,} x_{14i,} x_{15i}, x_{16i}, x_{17i}, x_{18i}, x_{19i}, x_{20i}$

$, x_{21i}, x_{22i}, x_{23i}, x_{24i}, x_{25i}, x_{26i}, x_{27i}, x_{28i}, x_{29i}, x_{30i})$ are simulated of a multivariate Gaussian distribution $X \sim N_{30}(\mu, \Sigma)$,nwhere μ is mean vector $\mu \epsilon R^n$ and Σ is the covariance matrix with $(\Sigma_x)_{ij} = (2^{-1})^{|i-j|}$. From the computed results from fourth simulation, we can see that our proposed method performs better than Bayesian and non- Bayesian methods (BAnet and crq) in terms of RMSR and SD criteria. The RMSR and SD generated by (New B L Tobit Q Reg) are much smaller compared with the RMSR and SD generated by the other two methods. Also, let us evaluate our proposed (New B L Tobit Q Reg) compared with the two other methods via coefficients estimation by direct way criterion. From the results which belong to estimation of direct way criterion, the coefficients estimation by our proposed method (New B L Tobit Q Reg) was very close to true parameters compared with Bayesian and non-Bayesian methods (BAnet and crq). This indicate that (New B L Tobit Q Reg) method has quite good performance compared with two other methods. Also to demonstrating the performance of our proposed method (New B L Tobit Q Reg), the real data approach has been used. The extramarital Affairs data is used for testing our proposed method (New B L Tobit Q Reg). This data is available in the R package "AER" and it was presented by (Fair in (1978))[21]. This data is used by (Ji et al., (2012))[37], (Alhamzawi, (2014))[2], and others and it contains censored response variable representing the number of times extramarital sexual encounters occurred during the past year (affairs). The eight explanatory variables are gender (1 for female and 2 for male), age, number of years married, children (2 for existence of children in the marriage and 1 without children), religiousness (scale from 1 to 5), level of education, how much prestige their occupation (scale from 1 to 7) and rating the happiness in their marriage (scale from 1 to 5). The sample size of extramarital Affairs data is 601 observations. The response variable (affairs) has the high data censored, where 451 observations are censored, and the rest of the observations uncensored (Fadel Hamid Hadi Alhusseini,2017)[25]. The (MSE) is computed by our proposed method and it is smaller than that of Banet method, for all Tobit quantile levels. These results prove that our proposed method can be considered better than Banet method.

From all results which are listed from simulation approaches and real data we see our proposed method (New B L Tobit Q Reg) has a quite good performance in coefficients estimation and variable selection in Tobit Q Reg model compared with other methods. Also it is consider a new extension to Bayesian penalized Tobit Q Reg model

For implementing variables selection by our proposed method (NewBL Tobit Q Reg) we will build a new MCMC algorithm for determining the probability value for each independent variable in our model. The coefficients estimated by Bayesian approach is implemented via thousands of iterations. At each iteration new estimator will be generated according to the proposed algorithm (Gramacy, R. B., & Lee, H. K. H. (2008))[32]. All these estimations are compared with interval (-0.05, 0.05). If the coefficients estimated which belong to the independent variables outside the open interval (-0.05, 0.05), at probability value is greater than 0.5. This means the independent variable has a high relative importance in the model. On the contrary, if the estimated coefficients which belong to independent variables of outside the open interval (-0.05, 0.05), at probability value are less than 0.5. This means the independent variable has a weak relative importance in model (Reed, C, (2011))[62] and (Fadel Hamid Hadi Alhusseini 2017)[27][28]. Our method is implemented on Extramarital Affairs data at four Tobit quantile levels ($\theta_1 = 0.15, \theta_2 = 0.35, \theta_3 = 0.75$ and $\theta_4 = 0.95$). In Tobit Q Reg model at Tobit quantile level$\theta_1 = 0.15$. There are four independent variables (x_1, x_2, x_5, x_8) have probability values greater than 0 and there are four independent variables (x_3, x_4, x_6, x_7) which have probability values less than 0.5. This means the independent variables (x_1, x_2, x_5, x_8) have a high relative importance and the independent variables (x_3, x_4, x_6, x_7) have a low relative importance in Tobit Q Reg at Tobit quantile level$\theta_1 = 0.15$.

At quantile level $\theta_2 = 0.35$, we see independent variables $(x_1, x_2, x_5, x_6, x_7, x_8)$ have a high relative importance in our model because of their probability values are exceed 0.5. But the independent variables (x_3, x_4) are have a low relative importance in the Tobit quantile regression model at level $\theta_2 = 0.35$. Because of their probability values, not exceed 0.5. At Tobit quantile level θ_3=0.75 and Tobit quantile level θ_4=0.95 all independent variables have a high relative importance in our models (Tobit Q Reg) . Because of their probability values greater than 0.5. More details about our proposed method (New BL Tobit Q Reg) are given in chapter three ,

We also have new contribution to coefficients estimation and variable selection in composite Tobit Q Reg model via our proposed method Bayesian composite Tobit quantile regression (Bayesian composite Tobit Q Reg). The aforementioned approaches for modeling Tobit Q Reg focus on a single quantile level. However, the efficiency of Tobit Q Reg estimators depends on the Tobit quantile level. Because the distribution is unknown, it is difficult to select the most informative Tobit quantile which can provides an efficient estimator. (Zou and Yuan (2008))[76] were proposed new method for estimating the coefficients in regression model called composite quantile regression (Composite Q Reg) , and show the relative efficiency of these estimators is greater than 70% when compare with least square estimator regardless of the error distribution. Composite quantile regression (Composite Q Reg) estimators are robust to the heavy tailed or outliers in the dependent variables and more efficient than a single quantile regression. For these characteristics we employ this approach in this study. The composite Tobit Q Reg model will be as follows:

$$T_i^* = \alpha_h + x_i^T \beta_h + \varepsilon_i, \qquad , \qquad y = \max(C, T_i^*) \qquad [22]$$

where $h = 1, \ldots\ldots, H,.$, H is different quantiles ,$0 < \theta_1 < \theta_2 < \cdots < \theta_H < 1$ and $i = 1, \ldots\ldots, n$. In composite Tobit Q Reg the parameters are estimated by solving the following equation

$$\left(\hat{\alpha}1, \hat{\alpha}2, \ldots, \hat{\alpha}\mathrm{H}, \hat{\beta}\right) = \underset{\alpha_1\ldots\ldots\alpha_{2,\beta}}{Min} \quad \sum_{h=1}^{H} \left\{ \sum_{i=1}^{n} \rho_{\theta_h} \left(y_i - \max(c, \alpha_h + x_i^T \beta)\right) \right\}, \qquad [23]$$

The Equation (23) is not differentiable at 0 point. Therefore, the minimization can be achieved through some modifications to algorithm proposed by (Koenker and D'Orey (1987))[46]. To estimate the Composite Tobit Q Reg coefficients, a Bayesian method is considered a new approach for this purpose. Via anew Gibbs sampler is proposed for posterior distribution inference. From known, the random error distribution of Tobit Q Reg is belong to (ALD). Therefore the joint distribution of response variable $[y^T_{(i=1,2,\ldots\ldots,n)}]$ given $[x^T_{(i=1,2,\ldots\ldots,n)}]$, $[\alpha^T_{(j=1,2,\ldots.h)}]$ and $[\beta^T_{(j=1,2,\ldots.h)}]$ for composite Tobit Q Reg is

$$f(y|X, \alpha, \beta) = \prod_{h=1}^{H} \theta_h^n (1 - \theta_h)^n exp\left\{ -\sum_{i=1}^{n} \rho_{\theta_h}(y_i - max\{c, \alpha_h + x_i^T \beta\} \right\} \qquad [24]$$

It is difficult to solve Equation (24) directly because of the mixture of H components. Following (Huang and Chen (2015))[34], we use a cluster assignment matrix C whose $(i, h)^{th}$element C_{ih} is equal to 1 if the i_{it} subject belongs to the h_{th} cluster, otherwise$C_{ih} = 0$. The element C_{ih} is treated as missing value. Thus, our likelihood takes the form "Fadel Hamid Hadi Alhusseini and Vasile Georgescu,2017"[23].

$$f(y|X, \alpha, \beta) = \prod_{h=1}^{H} \prod_{i=1}^{n} \left[\theta_h (1 - \theta_h) exp\left\{\rho_{\theta_h}(y_i - max\{c, \alpha_h + x_i^T \beta\}) \right\}\right]^{C_{ih}} \qquad [25]$$

Also when using the equation (25) directly leads to difficult MCMC algorithm. Now we will used the Kozumi and Kobayashi proposition via reformulation of ALD to a mixture of normal distributions. For our Bayesian hierarchical composite Tobit Q Reg, the formulation of (Kozumi and Kobayashi (2011))[49] can be written as:

$y_i|\alpha_h, \beta, z_i \sim Normal\ (max\{c, \alpha_h + x_i^T\beta\} + (1-2\theta_h)m_i, 2m_i)$, $where\ m_i \sim Exp\big(\theta_h(1-\theta_h)\big)$ .Under this formulation, the likelihood function of response variable is given as follows

$$f(T_i^*|\alpha_h, \beta, z_i) = \prod_{i=1}^{n} \frac{1}{\sqrt{4\pi m_i}}\ e^{\frac{1}{2}\sum_{i=1}^{n}\sum_{h=1}^{H}\frac{c_{ih}\,(T_i^*-\alpha_h-x_i^T\beta-(1-2\theta_h)m_i)^2}{2z_i}} \qquad [26]$$

The composite Tobit Q Reg parameters have desirable conditional conjugacy features for constructing a simple and attractive Gibbs sampler algorithm for fitting our model to the data. The equation (26) considers the likelihood function to coefficient estimation and variable selection of our proposed method (composite Tobit Q Reg) model. Therefore, we need hierarchical prior distributions to obtain full conditional posterior distributions of our proposed method. The hierarchical prior distributions of composite Tobit quantile regression will be as follows:

The prior distributions to parameters of (composite Tobit Q Reg) model is summarized as follows: The uniform distribution is assigned to parameter α_h , where $p(\alpha_h) \propto 1$ and normal prior distribution with mean 0 and variance s_j is belong to the parameter β_j . The parameter s_j is take exponential prior distribution with rate parameter (λ_j) . The parameter λ_j is distribute as gamma prior distribution with shape parameter (a) and scale parameter (b). a and b are two hyper parameters which take initial values, where a=0.1 and b= 0.1. Therefore our Bayesian hierarchical model will be as follows:

$$y_{i=max\{c,T_i^*\}}, \qquad i=1,\dots,n,$$

$$T_i^*|\alpha_h, \beta, z_i \sim [N(\alpha_h + x_i^T\beta + (1-2\theta_h)m_i, 2m_i)]^{c_{ih}},$$

$$p(\alpha_h) \propto 1 \qquad [27]$$

$$m_i \sim Exp\ (\theta_h(1-\theta_h))\ ,$$

$$\beta_{j\sim N(s_j)}$$

$$s_{j\sim} Exp\left(\frac{\lambda_j}{2}\right),$$

$$\lambda_j \propto \lambda_j^{a-1} exp\{-b\lambda_j\}.$$

From equation (26) and equation (27), we will obtain on the conditional posterior distributions. Therefore the hierarchical model to composite Tobit Q Reg model with Lasso penalty are:

The full conditional posterior distribution of latent variable (T_i^*) is given by

$$T_i^*|y_i, m_i, \alpha_h, \beta \sim \begin{cases} \{\Upsilon(y_i), & if\ y_i > c; \\ \left\{\prod_{h=1}^{H}[N(\alpha_h + x_i^T\beta + (1-2\theta_h)m_i, 2m_i)]^{c_{ih}}\right\} I(T_i^* \le c), otherwise \end{cases} \qquad [28]$$

where c is equal zero and $\Upsilon(y_i)$ denoted to a degenerate distribution. The full conditional posterior distribution of m_i for $i = 1, \dots, n,$ is the Inverse Gaussian with mean

$\sqrt{\sum_{h=1}^{H} C_{ih}/(T_i - \alpha_h - x_i^T\beta)^2)}$ and shape parameter $\sum_{h=1}^{H} C_{ih}/2$. The full conditional posterior distribution of α_h is normal distribution with mean $(\tilde{\sigma}_h^2 \sum_{i=1}^{n} C_{ih} (T_i^* - x_i^T\beta - (1-2\theta_h)m_i)/2m_i)$ and variance $(\sum_{i=1}^{n}(C_{ih}/2m_i))^{-1}$. The full conditional posterior distribution of β_j[j=1, 2,….k] is normal distribution with mean $(\tilde{\sigma}_j^2 \sum_{h=1}^{H}\sum_{i=1}^{n} C_{ih}x_{ij}(y_i - \alpha_h - \sum_{l\neq j} x_{il}\beta_l - (1-2\theta_h)m_i)/(2m_i))$ and variance $(\tilde{\sigma}_j^2 = (\sum_{i=1}^{n}\sum_{h=1}^{H} C_{ih}\, x_{ij}^2/2m_i) + s_j^{-1})^{-1})$. The full conditional posterior distribution of $s_i^{-1}[j = 1,2, \dots \dots k]$ is the inverse Gaussian with mean $\sqrt{\lambda_j/\beta_j^2}$ and shape parameter λ_j. The full conditional posterior distribution of λ_j is gamma distribution with shape parameter $(a + 1)$ and scale parameter $(b + \frac{s_j}{2})$. The full conditional posterior distribution of $C_{ih} = (C_{i1}, C_{i2}, \dots \dots, C_{iH})^T$ is a multinomial distribution; $p(C_i|y, X, \alpha, \beta, z) \propto \text{multinomial}(1, \hat{p}_1, \dots., \hat{p}_H)$, where $\hat{p}_H = \frac{\exp[-(y_i - \alpha_h - x_i^T\beta - (1-2\theta_h)m_i/2m_i]}{\sum_{h=1}^{H}\exp[-(y_i - \alpha_h - x_i^T\beta - (1-2\theta_h)m_i/2m_i]}$. From the full conditional posterior distributions are show in above formulas, we will obtain a simple and efficient Gibbs sampler algorithm. Our algorithm was run for 16,000 iterations and the first 1000 were removed as burn in. Then, we think the subsequent iterations by keeping every 5th simulation draw and discarding the rest. For assessing our proposed method (Bayesian Composite Tobit Q Reg) the simulation approach and real data has been used "Fadel Hamid Hadi Alhusseini and vasile Georgescu,2017"[23].

In simulation study, we will use three simulation approaches for assessing the performance of our method (Bayesian Composite Tobit Q Reg) via compared this method with Bayesian and non-Bayesian methods ‘crq, BAnet’. The methods under comparison are evaluated by (MMAD) where$MMAD = median(mean(\left|x^T\hat{\beta} - x^T\beta\right|))$. And the standard deviations (SDs) of the MADs are also presented.

Simulation 1 belongs to very sparse case. So, the true model parameters are $\beta = (1,0,0,0,0,0,0,0)^T$. The data is generated by the true model as follows:

$$y_i = \max(0, T_i^*)\,, \quad i = 1,2, \dots \dots \dots, 100\ , \qquad T_i^* = x^T\beta + \varepsilon_i$$

where X are distributed multivariate normal distribution a $N_k(0, \Sigma_x)$ with $(\Sigma_x)_{hl} = 0.5^{|h-l|}$, and(k=8) The T_i^* is latent variable with mean zero. The residuals are generated from 5 distributions: a $\varepsilon_i \sim N(0,1)$ (ε_i is distributed stander normal with mean zero and variance one) , a $\varepsilon_i \sim t_{(3)}$($\varepsilon_i$ is distributed a t-distribution at 3 degrees of freedom),a $\varepsilon_i \sim 0.5N(1,1) + 0.5N(-1,1)$($\varepsilon_i$ is distributed mixture normal distribution) a $\varepsilon_i \sim$ Laplace (0,1),(ε_i is distributed stander Laplace distribution with location parameter 0 and scale parameter 1) and $\varepsilon_i \sim 0.5\text{Laplace}(1,1) + 0.5\text{Laplace}(-1,1)$ (ε_i is distributed mixture Laplace distribution)

We set H=3 so that three Tobit quantile levels are:$\theta_1 = 0.25, \theta_2 = 0.50, and\ \ \theta_3 = 0.75$ where $\theta_h = h/(H+1)$, "Fadel Hamid Hadi Alhusseini and vasile Georgescu,2017"[23].

From the results, we see the MMAD and standard deviations (SDs) computed by our proposed method (Bayesian Composite Tobit Q Reg) are much smaller than the standard deviations (SDs) and MMAD computed by crq and BAnet for all error distributions. This means that our proposed method (Bayesian Composite Tobit Q Reg) has a better performance than the two other methods.

Simulation 2 is belong to dense case. So, the true model parameters are $\beta = \underbrace{(0.85, \dots \dots \dots., 0.85)}_{15}{}^T$. The data is generated by the true model as follows:

$$y_i = \max(0, T_i^*), \quad i = 1,2,\ldots\ldots.,100 \ , \qquad T_i^* = x^T\beta + \varepsilon_i$$

Also the covariates (X), distributed multivariate normal distribution a $N_k(0, \Sigma_x)$ with $(\Sigma_x)_{hl} = 0.5^{|h-l|}$, and(k=15) The T_i^* is latent variable with mean zero. The residuals are generated for all distributions under consideration. In this simulation the results show that our proposed method (Bayesian Composite Tobit Q Reg) performs better than two other methods at three Tobit quantile levels and all residuals distributions. This clear from the results of (SDs) and MMAD which are generated by the methods where, (SDs) and MMAD are generated by our proposed method smallest than (SDs) and MMAD are generated by two other methods.

Simulation 3 belongs to group structures Case. Therefore, the true model parameters are $\beta = ((1.5,1,0), (0,0,0), (1.8,1,0), (0,0,0), (1,1,0)^T$. The data is generated by the true model as follows:

$$y_i = \max(0, T_i^*), \quad i = 1,2,\ldots\ldots.,100 \ , \qquad T_i^* = x^T\beta + \varepsilon_i$$

Also the covariates (X), distributed multivariate normal distribution a $N_k(0, \Sigma_x)$ with $(\Sigma_x)_{hl} = 0.5^{|h-l|}$, and(k=15) The T_i^* is latent variable with mean zero. The residuals are generated for all distributions under consideration. From the results of MMAD and SD are generated by our proposed method (Bayesian Composite Tobit Q Reg) are smallest than the SD and MMAD are generated by BAnet and crq for all residuals distributions. These results indicate that the performance of our proposed method was well compared with two other methods (BAnet and crq). From the results which are recorded of three simulations, we conclude the our proposed method (Bayesian Composite Tobit Q Reg) has big importance for variable selection and coefficients estimation in Tobit quantile regression model compared with other methods in same field. To assess the performance of our method (Bayesian Composite Tobit Q Reg) with real data we will use labor force participation data which are available in the AER package in R. These data was introduced by (Mroz (1987))[56] and analysed using Tobit Q Reg in (Yu and Stander (2007))[]. The description of labour force participation data are contain response variable y: (*hours*) is consider wife's hours of work in 1975 and six explanatory variables are x_1: (*education*) wife's education in years, x_2: (*experience*) actual years of wife's previous labour market experience, x_3: (*age*) age, x_4: (*tax*) marginal tax rate facing the wife, x_5: (*oldkids*) the number of children between ages six and eighteen in household and x_6: (*fincome*) family income. the sample size of these data are n = 753 observations , from it 325 are censored observations and the rest (423) observations are uncensored observations. We will used four Tobit quantile levels ($H = 4, h = 1,2,3, H = 4$)). It is determined as follows: $\theta_h = h/(H+1)$.

where $\theta_1 = \frac{1}{5} = 0.20, \theta_2 = \frac{2}{5} = 0.40 \, , \theta_3 = \frac{3}{5} = 0.60 \ and \ \theta_4 = \frac{4}{5} = 0.80$.

Then $\theta \in \{0.20,0.40,0.60,0.80\}$.

For evaluating our proposed method (Bayesian Composite Tobit Q Reg) we compared it with two other methods (Banet, crq) the mean squared error (MSE) for all methods studied has been calculated. Through all Tobit quantile levels the (MSE) calculated by our proposed method (Bayesian Composite Tobit Q Reg) much smaller than (MSE) is calculated by two other methods. Therefore our proposed method (Bayesian Composite Tobit Q Reg) has a better performance compared with (Banet, crq) methods. From the results shown in simulations and real data approaches we conclude our proposed method (Bayesian Composite Tobit Q Reg) is quite well in coefficients estimation and variable selection in Tobit Q Reg model "Fadel Hamid Hadi Alhusseini and Vasile Georgescu, 2017"[23].

In our method Bayesian Composite Tobit Q Reg, we will propose a new MCMC algorithm to variable selection through determination of relative importance for explanatory variables in our model. Where, all previous methods in field of variable selection contain their algorithms on the following condition, if coefficients estimation are closed from zero put them zero exactly. In our MCMC algorithm, we will propose a new procedure in variable selection via determination of relative importance to each explanatory variable via compared Bayesian estimation for each explanatory variable with open interval (-0.05, 0.05), for computation the probability value for this explanatory variable. If the Bayesian estimation for this explanatory variable outside interval (-0.05, 0.05) by probability value greater than 0.5, then this explanatory variable is active in our model. On the contrary, if the Bayesian estimation for this explanatory variable outside interval (-0.05, 0.05) by probability value less than 0.5 then this explanatory variable is inactive in our model. Therefore, we can delete it from our model. See (Reed, C (2011))[62] and (Alhamzawi, R., (2016))[5], (Fadel Hamid Hadi Alhusseini, 2017))[27]. We employed our proposed method Bayesian Composite Tobit Q Reg to determinate the relative importance for explanatory variables of labour force participation data at two groups of Tobit quantile levels (H=5 and H=10). From results at five Tobit quantile levels (H=5) there are two explanatory variables (x_3: Age, x_5: Oldkids) have a high relative importance in our model. Because these explanatory variables have probability values greater than 0.5, there are four explanatory variables (x_1: Education, x_2: Experience, x_4:Tax and x_6:Fincome) with a low relative importance in our model (composite Tobit Q Reg at five composite quantile levels (H=5)). Also we used Bayesian Composite Tobit Q Reg at ten Tobit quantile levels (H=10). The results show four explanatory variables (x_1:Education,x_3:Age, x_4:Tax, x_5:Oldkids) with a high relative importance in our model at ten Tobit quantile levels, because of their probability values greater than 0.5. But the rest of explanatory variables (, x_2:Experience, x_6:Fincome) have a low relative importance in composite Tobit Q Reg model at ten Tobit quantile levels,, because of their probability values less than 0.5. Therefore, we can cancel explanatory variables(x_2: Experience, x_6: Fincome) from composite Tobit Q Reg model at ten Tobit quantile levels lines. More details are shown in chapter four ,the orginal results are published in [23]Fadel Hamid Hadi Alhusseini, and Vasile Georgescu 2017 " Bayesian composite Tobit quantile regression." Journal of Applied Statistics (2017): pp 1-13.
According to the results we think that our proposed methods (new Bayesian Lasso Tobit Q Reg and Bayesian Composite Tobit Q Reg) have a good performance in coefficients estimation and variables selection in Tobit Q Reg model.

Therefore, we will use these two proposed methods for modelling the relationship between response variable (Iraqi banks' investments) and nine of explanatory variables are x_1:Banking Deposits , x_2:Banking profits, x_3:Bank capital, x_1:Bank reserves, x_5:Banking Loans, x_6:Advertising Expenses, x_7:Age of the Bank, x_8:Number of Bank Branches, x_9:bad debt. The response variable (Iraqi banks' investments) is censored from left side at zero point. Tobit regression model considers effective regression model with censored response variable. But Tobit regression model is sensitive from many problems. Also Tobit regression model is not capable of providing a complete information about the stochastic relationships between dependent variable and explanatory variables. To overcoming these problems the Tobit quantile regression (Tobit Q Reg) model has been used. In order to analyse our data (banking investments data), we will use Tobit quantile regression model as follows:

$$y_i = \max(0, T_i^*) \qquad ,T_i^* = \alpha + \beta_{1\theta}x_{1j} + \beta_{2\theta}x_{2j} + \beta_{3\theta}x_{3j} + \beta_{4\theta}x_{4j} + \beta_{5\theta}x_{5j} + \beta_{6\theta}x_{6j} + \beta_{7\theta}x_{7j} + \beta_{8\theta}x_{8j} + \beta_{9\theta}x_{9j} + u_{i\theta} \qquad {}_{j=1,2,\ldots\ldots 47}$$

where: y is censored response variable (Iraqi banks' investments). T_i^* is latent variable $x_i, [i = 1,2, \ldots 9]$ are explanatory variables as above description. $(\beta_{1\theta}, \ldots, \beta_{9\theta})$ parameters which are estimated by our proposed methods as following:

New Bayesian Lasso Tobit Q Reg is used for coefficients estimation and variable selection in Tobit Q Reg model which is showed above via thirty Tobit quantile levels. This mean we will obtain thirty Tobit quantile regression models. We employed our proposed method (New Bayesian Lasso Tobit Q Reg) in two sides:

Firstly: coefficients estimation of Tobit quantile regression model via thirty Tobit quantile levels.

Secondly: we will determine the relative importance to explanatory variables in Tobit quantile regression model via the probabilistic approach. If the explanatory variable has a probability value greater than 0.5 this means it has a high relative importance in constructing our model. But when, its probability value less than 0.5. This means it has a negligible relative importance in constructing our model. Therefore, we can remove it from this model. To identify active independent variables in Iraqi banks investments which depend on the relative importance to these variables thirty different Tobit quantile levels have been used.

Our proposed method (Bayesian composite Tobit Q Reg) has a high quality for coefficients estimation and variable selection in composite Tobit Q Reg model Here we will use six groups of Tobit quantile levels , (five Tobit quantile levels, ten Tobit quantile levels, fifteen Tobit quantile levels, twenty Tobit quantile levels, twenty five Tobit quantile levels and thirty Tobit quantile levels). Each group has a specific composite Tobit Q Reg model based on the number of Tobit quantile levels. We will use our method (Bayesian composite Tobit Q Reg) in two ways as follows:

Firstly: coefficients estimation in composite Tobit Q Reg model at six groups of Tobit quantile levels:

Secondly: variable selection in composite Tobit quantile regression model at six groups of Tobit quantile levels.

General Concepts

1.1:Variable Selection

In many applications, variable selection (VS) has become a popular. It is the process of choosing a subset of the significant variables for use in model constructing. It provides a good prediction as well as highlighting the variables, which are significant in fitting the model (Griffin and Brown, (2010))[29]. The main assumption when using VS is that the data contains many unimportant variables. Therefore, there are two main objectives must distinguished when create regression model: The first objective is prediction accuracy with a regard of a good structure of the regression model . The second objective is explanation for the coefficients of regression, through the attempt to determine influential independent variables and obtain insight on the relevance between the independent variables and the response variable via the structure of regression model. In fact, many independent variables may be considered as a weak independent variables in the model , but only a little number will have a significant impact. But identify insignificant independent variables are hard matter. Oftentimes, variable selection considers a good practical advantage and one of important requirements when constructing the regression model. Therefore, excluding the independent variables have an insignificant effect on regression model are of the interest of the model . And keeping the important independent variables in model , this it is improve the predictive accuracy in this model, also this is useful in interpretation.
In our thesis, we focus on extension Lasso technique in Bayesian approach.

1.1.1: Classical Lasso Regression

Lasso Regression is a method for coefficients estimation and variable selection concurrently. The word Lasso is abbreviated for a set of words "Least Absolute Shrinkage and Selection Operator" . This method is introduced by (Tibshirani (1996))[64]. The Lasso regression coefficients are given by:

$$:\hat{\beta}^{lasso} = \text{minimize} \sum_{i=1}^{n}\left(y - X\hat{\beta}\right)^2 \quad s.t \ \sum_{j=1}^{p}\left|\beta_j\right| \leq t \qquad [1.1]$$

where $\sum_{j=1}^{p} |\beta_j| \leq t$ $\quad L_{1-}$ *norm for regreesion coefficents.*

t is tuning parameter which is responsible about quantity of shrinkage .

1.1.2: Variable Selection in Tobit Quantile Regression Model

1.1.2. 1: Lasso Tobit Quantile Regression Model

Tobit QReg model is consider on of important regression models is defined according the equation (1.9). But the Lasso Tobit quantile regression model take the following formula

$$\min_{\alpha_\theta,\beta_\theta} \sum_{i=1}^{n} \rho_\theta \left(y_i - max\{C, y_i^*\}\right) + \lambda \sum_{j=1}^{p} \left|\beta_j\right| \qquad [1.2]$$

where quantity $\lambda \sum_{j=1}^{p}\left|\beta_j\right|$, is called penalty Lasso. The equation 1.2 can achieving variable selection and coefficient estimation in TQReg model simultaneously.

1.1.2.2: Bayesian Lasso Tobit Quantile Regression Model

It is consider efficient and effective method for achieving variable selection and coefficients estimation in Tobit quantile regression model .(Tibshirani (1996))[64] mentioned that Bayesian Lasso regression model is achieved through using Laplace prior distribution. The Bayesian Lasso Tobit quantile regression can be obtaining via multiply Likelihood asymmetric Laplace distribution (ALD) by prior Laplace distribution. From known the prior distribution plays an important role for determined appropriate method for variable selection (Fadel Hamid Hadi Alhusseini, 2017)[25]. When we are used The normal distribution for parameters vector β_θ, as prior

$$p(\beta|\mu,\vartheta) = \frac{1}{\sqrt{2\pi\vartheta}} e^{-\frac{(\beta_\theta-\mu)^2}{2\vartheta}} \qquad [1.3]$$

the posterior distribution are took the following formula :

$$f(\beta_\theta|y,x,\mu,\vartheta) \propto exp\left\{-\sum_{i=1}^{n} \rho\theta(y_i - max\{0, y_i^*\}\right\} - \frac{(\beta_\theta-\mu)^2}{2\vartheta} \qquad [1.4]$$

Bayesian estimation in equation (1.4) is a Bayesian ridge regression. Hence, all coefficients will shrinkage to zero but not equal zero exactly. But when we employ the Laplace prior distribution for parameter vector β_θ, it which is take the following equation:

$$g(\beta_\theta|\lambda) = (\lambda/_2)^p \exp\left(-\lambda \textstyle\sum_{j=1}^{p} |\beta_\theta|\right) \qquad [1.5].$$

We will obtained the posterior distribution for the Tobit quantile regression coefficients: as follows

$$f(\beta_\theta|y,x,\lambda) \propto exp\left\{-\sum_{i=1}^{n} \rho\theta(y_i - max\{0, y_i^*\}\right\} - \lambda\sum_{j=1}^{p}|\beta_\theta| \qquad [1.6]$$

Bayesian estimation in Equation (1.6) is the Bayesian Lasso Tobit Q Reg. Hence, some parameters will shrinkage to zero exactly, this means, Bayesian Lasso regression achieves variable selection and coefficient estimation in Tobit Q Reg model.

The Bayesian Lasso Tobit Q Reg model is used the ALD as Likelihood function (Yu and Moyeed, (2001))[]. And it is uses the Laplace prior distribution for achieves the variable selection in Tobit Q Reg model. But It is difficult to dealing with Laplace distribution directly, therefore, many researchers use a simple formula for Laplace distribution which introduced by (Andrews and Mallows (1974)) [5].

$$\frac{\lambda}{2} e^{-\lambda|\beta_j|} = \int_0^\infty \frac{1}{\sqrt{2\pi s_j}} e^{\frac{-\lambda^2}{2s_j}} \frac{\lambda^2}{2} e^{\frac{-\lambda^2}{2} s_j} \, ds_j \qquad \lambda > 0 \qquad [\ 1.7\ ]$$

This formula in equation (1.7), which product active and simple Gibbs sampling algorithm in field of Bayesian variable selection.

1.1.2. 3.: Bayesian Tobit Quantile Regression With New Lasso

Lasso technique is used with a Bayesian approach when the parameter vector belongs to Laplace prior distribution (Tibshirani (2011))[65] and (Park and Casella (2008))[60]. They used another formula for Laplace distribution, this formula is scale mixture of normal (SMN) and is consider more flexibilities for achieving variable selection which it proposed by (Andrews and Mallows: (1974))[5]. Recently ,(Mallick and Yi (2014))[55] provided a new formula to Laplace prior distribution by using the SMU via following formula.

$$\frac{\lambda}{2}e^{-\lambda|\beta|} = \int_{u>|\beta|}^{\infty} \frac{1}{2u}\,\frac{\lambda^2}{\Gamma(2)}u^{2-1}e^{-\lambda u}\,du \qquad \lambda > 0 \qquad [1.8]$$

where $(\frac{1}{2u})$ is (pdf) for uniform prior distribution and rest function from above formula it represent (pdf) for gamma prior distribution with rate parameter two and scale parameter λ . and $\Gamma(2) = (2-1)! = 1$

prove

$$\frac{\lambda}{2}e^{-\lambda|\beta|} = \int_{s>|\beta|}^{\infty} \frac{\lambda^2}{2}e^{-\lambda s}\ \ ds \qquad [1.9]$$

$$\frac{\lambda}{2}e^{-\lambda|z|} = \left[\frac{\lambda^2}{2}\frac{e^{-\lambda s}}{-\lambda}\right]_{|\beta|}^{\infty} = \left[-\frac{\lambda}{2}e^{-\lambda s}\right]_{|\beta|}^{\infty} = -\frac{\lambda}{2}e^{-\lambda\infty} - \left(-\frac{\lambda}{2}e^{-\lambda|\beta|}\right) = \frac{\lambda}{2}e^{-\lambda|\beta|}$$

According proposition of (Mallick and Yi (2014))[55] the equation (1.9) which representation alternative formula for Laplace prior distribution for implementation variables selection in classical liner regression model. In this thesis, we will use the equation (1.9) in our proposed method (new Bayesian Lasso Q Reg) via building of an efficient and simple Gibbs sampling algorithm for achieving variable selection and coefficients estimation in Q Reg model. Also, we extend this idea in our proposed method (new Bayesian Lasso Tobit Q Reg) via constructing an efficient Gibbs sampling algorithm for achieving variable selection and coefficients estimation in Tobit Q Reg model.

Chapter 2

New Bayesian Lasso quantile regression

2.1- Introduction

The linear Q Reg model assumes that the outcome (response variable) (y_i) can be written as:

$$y_i = x_i^T \beta_\theta + \varepsilon_i, \qquad \theta \in (0,1), \qquad [2.1]$$

where x_i^T is a $1 \times k$ vector of covariates (explanatory variables), β_θ is a $k \times 1$ vector of unknown quantities and θ is the quantile level. Here, ε_i is the residual term whose density is restricted to have the θth quantile equal to 0. Similar to the standard mean regression, Q Reg aims at evaluating the conditional quantiles of the outcome of interest (y_i) given an explanatory vector x_i. It can be proved (Koenker and Bassett, (1978))[38] that the Q Reg coefficients of β_θ can be estimated by:

$$\min_{\beta_\theta} \sum_{i=1}^{n} \rho_\theta(y_i - x_i^T \beta_\theta) \qquad [2.2]$$

where $\rho_\theta(s)$ is the check function defined by $\rho_\theta(s) = s\{\theta - \mathrm{I}(s \le 0)\}$. Also, the check function can be written by another formula, as follows.

$$\rho_\theta(\varepsilon) = \begin{cases} \theta\varepsilon & if\ \varepsilon \ge 0 \\ -(1-\theta)\varepsilon & if \varepsilon < 0 \end{cases} \qquad [2.3]$$

Since equation (2.2) is not differentiable at the origin, there is no exact form solution for (2.2) (Koenker, (2005))[42] the minimization of (2.2) can be achieved by a linear programming algorithm (Koenker and D'Orey,(1987))[46].

Although asymptotic properties for Q Reg are well studied, the development of appropriate inference procedures has been difficult. For simplicity of notation, we will omit θ from the notation β_θ in the remainder of this chapter. One significant issue in building a regression model is the selection of the active regressors (covariates).. The selection process aims to increase the prediction accuracy and to get high interpretation (Alhamzawi et al., (2012))[1]. Nowadays, there has been considerable attention on sparse methods that include all regressors in the model and use informative priors to shrink inactive regression coefficients to zero. For example, Lasso (Tibshirani, (1996))[64], the adaptive Lasso (Zou, (2006)[], Dantzig selector (Candes and Tao, (2007))[15], matrix completion (Candes and Recht, (2009))[16], compressive sensing (Baraniuk, (2007))[10], Lasso Q Reg (Li and Zhu, (2008))[53] and adaptive Lasso Q Reg (Wu and Liu, (2009))[67]. A comprehensive account of these and other recent methods can be found in (Tibshirani, (2011))[65]. Similarly, from a Bayesian framework, (Park and Casella (2008))[60] proposed Bayesian Lasso for linear regression models by specifying scale mixture of normal (SMN) prior distributions on the regression coefficients and independent exponential prior distributions on their variances. (Sun et al. (2010))[63] developed Bayesian adaptive Lasso by allowing different shrinkage parameters for different coefficients. Based on the latest approaches Li et al. (2010) suggested Bayesian Lasso Q Reg and(Alhamzawi et al. (2012))[1] proposed Bayesian adaptive Lasso Q Reg. Some further extensions of the Lasso Q Reg models have also suggested by (Benoit et al. (2013))[13], (Alhamzawi and Yu (2014))[2] and (Alhamzawi (2014))[3], among others. Compared to the frequentist counterparts, the Bayesian models usually offer a valid measure of standard error based on a MCMC. It also offers a convenient method of incorporating regression coefficients uncertainty into predictive inferences. Moreover, the Bayesian formulation offers a flexible way of estimating the penalty parameter along with regression coefficients.

Our objective is to develop a Bayesian formulation for regularization in linear Q Reg. Recently, for linear regression, (Mallick and Yi (2014))[55] provided a different approach of Lasso-based model by employing the scale mixture of uniform formulation of the Laplace density. The performance of this method was illustrated via simulation studies and a real dataset. (Mallick and Yi (2014))[55] show that the new method performs quite well compared to some other existing methods. In this chapter , we propose a new formulation for Bayesian Lasso Q Reg by employing the scale mixture of uniform formulation. Then we develop a fully Bayesian treatment that leads to a simple and efficient Gibbs sampling algorithm with tractable full conditional posterior distributions.

2 .2– Estimation Methods

2.2.1- Bayesian Quantile Regression Model

Within the Bayesian Q Reg formulation, a popular choice of the error distribution has been skewed Laplace distribution (SLD); see Yu and Moyeed (2001). The probability density function (pdf) of a SLD is

$$f(\varepsilon_i|\sigma) = \theta(1-\theta)\sigma\, exp\{-\sigma\rho_\theta(\varepsilon_i)\}, \qquad \varepsilon_i \in \mathbb{R}, \qquad [2.4].$$

With mean $\mathrm{E}(\varepsilon_i) = \frac{(1-2\theta)}{\sigma\theta(1-\theta)}$, and $\mathrm{Var}(\varepsilon_i) = \frac{(1-2\theta+2\theta^2)}{\sigma^2\theta^2(1-\theta)^2}$.

where σ is the scale parameter $\sigma > 0$, and θ is shape parameter determines the quantile level in response distribution . It belongs to interval (0,1) (Yu and Zhang,(2005))[72]. The joint distribution of $\varepsilon = (\varepsilon_1, \varepsilon_2, \dots . \varepsilon_n)^{\boldsymbol{T}}$ is

$$f(\varepsilon_i|\sigma) = \theta^n(1-\theta)^n\sigma^n \exp\left\{-\sum_{\mathbf{i=1}}^{\mathbf{n}} \sigma\rho_\theta(\varepsilon_i)\right\}, \qquad [2.5]$$

Then

$$f(y|\sigma) = \theta^n(1-\theta)^n\sigma^n \exp\left\{-\sum_{\mathbf{i=1}}^{\mathbf{n}} \sigma\rho_\theta(y_i - x_i^T\beta_\theta)\right\}, \qquad [2.6]$$

Following(Koenker and Machado (1999))[48] the check (loss) function (2.2) is closely equivalent to (2.4). In particular, maximizing (2.4) is equivalent to minimizing (2.2). The relationship between (2.2) and (2.4) can be employed to represent the Q Reg method in the likelihood framework. The skewed Laplace distribution (SLD) has good properties, see (Yu and Zhang; (2005))[72] for more details. But the using of equation (SLD) directly is a hard task, and provide us difficult and inefficient algorithm. Also skewed Laplace distribution take various mixture representation (Kotz et al (2001))[44] . Therefore (Kozumi and Kobayashi (2011))[49] provided a Bayesian approach for Q Reg by reformulated the SLD as SMNs family. More specifically, let $\varepsilon_i \sim \mathrm{N}((1-2\theta)m_i, 2\sigma^{-1}m_i)$ Then the SLD for ε_i Arises when $m_i \sim Exp(\theta(1-\theta)\sigma)$. Here we use another formula to the random error ε_i As scale mixture normal distribution with mean $(1-2\theta)m_i$ And variance $2\sigma^{-1}m_i$.

where m_i Is distributed exponential distribution with rate parameter $(\theta(1-\theta)\sigma)$ and ϵ_i is distributed as normal distribution with mean (0) and variance (1).

Therefore $y_i \sim \mathrm{N}(x_i^T\beta_\theta + (1-2\theta)m_i, 2\sigma^{-1}m_i)$

Under the above hierarchical formulation, the posterior distribution of interest $p(\beta_\theta|\sigma, m_1, \dots \dots m_n)$ is a multivariate normal.

2.2.2. Bayesian Q Reg with the Lasso penalty

The Lasso quantile regression (Q Reg) (Li and Zhu, (2008))[53] is given by

$$\min_{\beta_\theta} \sum_{\boldsymbol{i=1}}^{\boldsymbol{n}} \rho_\theta(y_i - x_i^T\beta_\theta) + \lambda \parallel \beta_\theta \parallel, \qquad [2.8]$$

where λ, (($\lambda \geq 0$) is the shrinkage parameter**,** Also the equation (2.8) is not differentiable at 0, but possible, achieving parameters estimation through (rq.fit.Lasso) function (Koenker, R. (2005))[42] within Package ‘quantreg’ (2013).

The method of the Bayesian quantile regression model using coefficients estimation only. As method proposed by (Yu and Moyeed ,(2001))[]. But Bayesian Lasso quantile regression implements variables selection and coefficient estimation to quantile regression model simultaneously. (Li et al. (2010))[52] proposed the Bayesian Lasso for linear Q Reg model by putting a Laplace prior takes the form. $p(\beta_j|\lambda,\sigma) = {}^{\sigma\lambda}/_2\ exp\{-\sigma\lambda|\beta_j|\}$ On the j_{th} Q Reg coefficient. More specifically, they put a scale mixture of normal prior distributions on β_θ And exponential prior distributions for the variance parameters assuming they are independent. In this chapter, we put a Laplace prior on β_j Takes the form $p(\beta_j|\lambda) = {}^{\lambda}/_2\ exp\{-\lambda|\beta_j|\}$ And develop an alternative hierarchical Bayesian model of the Lasso model.

Following (Mallick and Yi, (2014))[55], the Laplace prior to β_j Can be written as:

$$\frac{\lambda}{2}e^{\{-\lambda|\beta_j|\}} = \frac{\lambda}{2}\int_{u_j>|\beta_j|} \lambda\ exp\{-\lambda u_j\}\,du_j \quad ,$$

$$= \int_{-u_j<\beta_j<u_j} \frac{1}{2u_j}\frac{\lambda^2}{\Gamma(2)}{u_j}^{2-1}\ exp\{-\lambda u_j\}\,du_j \quad , \qquad [2.9]$$

where $\Gamma(2) = (2-1)! = 1$

$$= \int_{-u_j<\beta_j<u_j} \frac{\lambda^2}{2}\ exp\{-\lambda u_j\}\,du_j \quad ,$$

where u_j is a mixing variable. We further put Gamma priors on, σ *and* λ. Using (2.7) and (2.9), our Bayesian hierarchical model can be formulated as follows:

$$\begin{aligned}
&y_i|\beta_\theta,\sigma,m_i \sim N(x_i^t\beta_\theta + (1-2\theta)m_i, 2\sigma^{-1}m_i),\\
&m_i|\sigma \sim exp\{\theta(1-\theta)\sigma\},\\
&\beta_j|u_j \sim Uniform\,(-u_j,u_j),\\
&u_j|\lambda \sim Gamma(2,\lambda)\ , \qquad [2.10]\\
&\sigma \sim \sigma^{a-1}exp(-b\lambda),\\
&\lambda \sim \lambda^{c-1}exp(-d\lambda)\ .
\end{aligned}$$

Here, $Exp(\theta(1-\theta)\sigma)$Refers to the exponential distribution with rate parameter $\theta(1-\theta)\sigma$.

2.3. Conditional Posterior Inference

The conditional posterior distributions are considered an important part in the Bayesian approach. Where the posterior distribution denoted $f(\beta|y)$ Gives us complete information about parameter estimation. The posterior distribution output from the likelihood function denoted $f(y|\beta)$ Provides full information about the data and prior distribution denoted $g(\beta)$ Which provides complete information about the unknown parameter: (Arto Luoma, (2014))[7] The conditional posterior distributions are defined according to a following mathematical formula:

$$f(\beta|y) = \frac{f(y,\beta)}{f(y)} = \frac{f(y|\beta)*g(\beta)}{f(y)} = \propto f(y|\beta)\ g(\beta). \qquad [2.11]$$

where :$\propto$ is (proportional to) represent constant values

Bayesian approach is an efficient method for estimation of coefficient quantile regression through deriving the good conditional posterior distribution. We will obtain conditional posterior distribution of $\beta, m, (m_1, \dots m_n)^T, u = (u_1, \dots u_n)^T$ and λ can be updated using an efficient MCMC-based computation technique.

The conditional posterior distribution of β is truncated normally with mean $\left(\sigma_{\beta_j}^2 \sum_{i=1}^{n} \frac{\sigma x_{ij}\left(y_i-(1-2\theta)m_i-\sum_{,j\neq l}^{p} x_{ij}\beta_j\right)}{2m_i}\right) I\{|\beta_j| < u_j\}$ and variance $\left(\sum_{i=1}^{n} \frac{\sigma x_{ij}^2}{2m_i}\right)^{-1}$. The conditional posterior distribution of m is inverse Gaussian distribution with mean $1/\sqrt{(y_i - x_i^t\beta)^2}$ and shape parameter $(\frac{\sigma}{2})$. Conditional posterior distribution of u_j is a left-truncated exponential distribution given by $u_j|\beta,\lambda \sim Exp(\lambda) I\{u_j > |\beta_j|\}$. Conditional posterior distribution of the penalty parameter λ is Gamma distribution with shape parameter $(c + 2p)$ and rate parameter $(d + \sum_{j=1}^{p} |\beta_j|)$. Also the conditional posterior distribution of σ is Gamma distribution with shape parameter $(a + \frac{3n}{2})$ and rate parameter $(\sum_{i=1}^{n}\left(\frac{(y_i - x_i^t\beta + (1-2\theta)m_i)^2}{4m_i} + \theta(1-\theta)m_i\right) + b)$. From the full conditional posterior distributions we proceed to sample each unknown parameter $(\beta,,, m, (m_1, \dots m_n)^T, u = (u_1, \dots u_n)^T$, λ and σ. We will obtaine a good Gibbs sampler for coefficient estimations and variable selection in new Bayesian Lasso quantile regression model.

2.5.Chapter Summary

In this chapter, we propose a new Bayesian Lasso quantile regression method for variable selection, assigning independent scale-mixture of uniform distributions for the regression coefficients. Then, a simple and efficient MCMC algorithm was presented for Bayesian sampler. Simulation studies and a real data set are used to investigate the performance of the proposed method compared to some other existing methods. Both simulated and real data examples show that the proposed method performs quite well compared to the other methods under a variety of scenarios.

Chapter three

New Bayesian Lasso in Tobit quantile regression

3.1: Introduction

The using of the optimal model is a challenging task, but for each dataset, there is an optimal model. In case of the Tobit regression model, there is an adaptation with left censored data. Since the seminal work of (Tobin, J., (1958))[66] this model has given good estimates when achieving the normal assumptions or when the datasets are empty from outliers. It becomes useless when one of the normal assumptions is broken or when outliers exist in the dataset. For overcoming this problem, Tobit quantile regression has been used, which can estimate its coefficient when the data are not achieving the normal assumption. This model was introduced by (Powell, (1986))[59] and is defined by the formula below:

$$T_i^* = \alpha_\theta + x_i^T \beta_\theta + \varepsilon_i, \qquad , \qquad y = \max(C, L_i^*) \qquad [3.1]$$

where T_i^* is latent unobserved response variable, $(\alpha_\theta, \beta_\theta)$ are intercept and vector unknown parameters of the Tobit quantile regression respectively, and θ is quantile level belonging to the open interval (0,1) as $\theta \in (0,1)$, y_i is censored response variable at the censoring point (C).

For estimating the parameters of the Tobit quantile regression, we minimized the following loss function:

$$\underset{\alpha_\theta, \beta_\theta}{min} = \sum_{i=1}^{n} \rho_\theta \left(y_i - max\{C, T_i^*\}\right) \qquad [3.2]$$

In Tobit quantile regression, the censored point (C) is equal to zero. Therefore, the loss function (3.2) takes the following formula:

$$\underset{\alpha_\theta, \beta_\theta}{min} = \sum_{i=1}^{n} \rho_\theta \left(y_i - max\{0, T_i^*\}\right) \qquad [3.3]$$

where $\rho_\theta(\varepsilon)$ is called check (loss) function of (Koenker and Bassett (1978))[38] at a quantile θ.
Therefore, the equation [3.3] can be rewritten as below:

$$\rho_\theta(u) = \begin{cases} (\theta)|\, y - max(0, \alpha_\theta + x_i^T \beta_\theta + \varepsilon_i)| & y \geq max(0, \alpha_\theta + x_i^T \beta_\theta + \varepsilon_i) \\ -(1-\theta)|\, y - max(0, \alpha_\theta + x_i^T \beta_\theta + \varepsilon_i)| & y < max(0, \alpha_\theta + x_i^T \beta_\theta + \varepsilon_i) \end{cases} \qquad [3.4]$$

The coefficients estimation (Tobit QReg) is performed by minimization of the equation [3.4]. The equation [3.4] is not differentiable at 0, so there is not an exact form of the solution for the parameters (Koenker, 2005))[42]. The minimization of the equation [3.4] can be resolved by a linear programming algorithm (Koenker and D'Orey, (1987))[46]. Although asymptotic properties for Tobit QReg are well studied and many algorithms are proposed, most of these algorithms are inefficient, when the response variable has high censored data. Currently, a possible estimation of coefficients (Tobit QReg) by (crq) function, exists in the package (quanTobit Req) (Koenker, (2011))[45].

3.2:Methodology of New Bayesian Lasso Tobit Quantile Regression

The Bayesian approach is considered one method for coefficients estimates in regression models. Where, it depends on the likelihood function for random error term and prior distribution for model parameters, as following:

$$p(\theta|Y_i) \propto f(Y_i|\theta) \times p(\theta) \quad [3.5]$$

$$posterior\ distribution \propto likelihood\ function \times prior\ distribution$$

3.2.1.Likelihood Function of Tobit Quantile Regression

(Koenker (2005))[42] provided us with a number of algorithms used to estimate Tobit QReg. Some of algorithms are inefficient, when the response variable dataset has a high amount of censored data. For instance linear programming algorithm (Koenker and D'Orey, (1987))[46] and others methods. Recently, (Yu and Stander, (2007))[] have proposed the Bayesian approach for estimating (Tobit QReg) even with high amount of censored Data. They were inspired by the suggestion of (Konker and Machado (1999))[48] and (Yu and Moyeed (2001))[]. These researchers observed convergence between loss function (3.4) and skew-Laplace distribution (SLD) (asymmetric Laplace distribution). Therefore, the random error term ε_i distributed as SLD with probability density function (pdf), takes the following formula:

$$f(\varepsilon_i|\mu,\sigma,\theta) = \frac{\theta(1-\theta)}{\sigma}\ \exp{-\rho_\theta}\left\{\left(\frac{\varepsilon_i-\mu}{\sigma}\right)\right\} \quad [3.6]$$

If $\mu = 0$ and $\sigma = 1$ then, the probability density function (pdf) to ε_i is:

$$f(\varepsilon_i|\sigma,\theta) = \theta(1-\theta)\exp(-\rho_\theta\{(\varepsilon_i)\}) \quad [3.7]$$

With mean, $E(\varepsilon_i) = \frac{1-2\theta}{\theta(1-\theta)}$ and variance, var $(\varepsilon_i) = \frac{1-2\theta+2\theta^2}{\theta^2(1-\theta)^2}$

$\rho_\theta(.)$ is the check(loss) function defined as in equation [3.7]. The joint distribution of $y = (y_1,\dots,y_n)^T\ given\ X = (x_1,\dots,x_n)^T$ is:

$$(y|X,\alpha,\beta,\sigma,\theta) = \theta^n(1-\theta)^n exp\left\{-\sum_{i=1}^{n}\rho\theta(y_i - max\{0,\alpha_\theta + x_i^T\beta_\theta + \varepsilon_i\}\right\} \quad [3.8]$$

where the likelihood function to the probability density function (pdf) of (SLD) with scale parameter equal one is:

$$f(y|X,\alpha,\beta,\sigma,\theta) = \theta^n(1-\theta)^n exp\left\{-\sum_{i=1}^{n}\rho\theta(y_i - max\{0,\alpha_\theta + x_i^T\beta_\theta + \varepsilon_i\}\right\} \quad [3.9]$$

Minimizing the equation [2.4] is equivalent to maximizing the likelihood function of the equation [3.9]. By using SLD directly, leads to hard computations, therefore (Kozumi and Kobayashi, (2011))[49] claimed the SLD can be reformulated as function scale mixture normal distribution. The likelihood function of equation [3.9] is possible to be rewritten as the following equation, according to suggestion of (Kozumi and Kobayashi) :

$$y_{i=max\{0,T_i^*\}},\qquad i=1,\dots,n,$$

$$T_i^*|\alpha_\theta,\beta_\theta,m_i \sim N\left(\alpha_\theta + x_i^T\beta_\theta + (1-2\theta)m_i, 2m_i\right) \quad [3.10]$$

Under the proposed method of (Kozumi and Kobayashi, (2011))[49] , rewriting the SLD for the errors as a SMN

$$f(T_i^*|\alpha_\theta, x_i^T, \theta, \beta_\theta, m_i) = \frac{1}{\sqrt{4\pi m_i}} e^{\frac{-\left(T_i^*-\alpha_\theta-x_i^T\beta_\theta-(1-2\theta)m_i\right)^2}{4m_i}}$$

The likelihood function of the probability density function ($f(T_i^*|\alpha_\theta, \beta_\theta, m_i)$) is

$$f(T_i^*|\alpha_\theta, x_i^T, \theta, \beta_\theta, m_i) = \left[\frac{1}{\sqrt{4\pi m_i}}\right]^n e^{-\sum_1^n \frac{\left(T_i^*-\alpha_\theta-x_i^T\beta_\theta-(1-2\theta)m_i\right)^2}{4m_i}} \quad [3.11]$$

where m_i is distributed experiential distribution with rate parameter $\theta(1-\theta)$. The equation [3.19] is an important part for constructing Gibbs samplers of posterior distributions for coefficient estimates of Tobit quantile regression model.

3.2.2. Bayesian Hierarchical of Prior Distributions

(Park and Casella (2008))[60], implement the Bayesian Lasso in traditional regression model by assigning Laplace priors for regression coefficients. Where, the Laplace distribution has the probability density function with $\sigma = 1$ as below:

$$f(\beta_j|\lambda) = \lambda/_2 \; e^{-\lambda|\beta_j|} \quad [3.12]$$

where λ is the shrinkage parameter and ($\lambda \geq 0$).

More researchers discussed Bayesian regularized Tobit quantile regression. For instance, (Yue and Hong 2012))[73], proposed Bayesian Tobit QReg with the group Lasso penalty, (Alhamzawi, (2013))[4], proposed the adaptive Lasso in Tobit QReg by using Bayesian technique. Also (Alhamzawi, (2014))[3] proposed a Bayesian elastic net penalty in Tobit QReg. Where most last methods using another picture from Laplace prior is a Scale Mixture Normal (SMN), according to (Andrews and Mallows (1974))[5]. It takes following manner: $\frac{\lambda_j}{2} e^{-\lambda_j|\beta_j|} = \int_0^\infty \frac{1}{\sqrt{2\pi s_j}} e^{\left(-\frac{\beta_j^2}{2s_j}\right)} \frac{\lambda_j^2}{2} e^{\left(-\frac{s_j\lambda_j^2}{2}\right)} ds_j$ Hence, Laplace prior can be rewritten as a function from two parts. The first part can be assigned to prior distribution for β_j,which distributes normally with mean zero and variance (s_j) as follows:

$$p(\beta_j|s_j) = \frac{1}{\sqrt{2\pi s_j}} exp\left\{-\frac{\beta_j^2}{2s_j}\right\} \quad [3.13]$$

where β_j unknown variance is s_j . The exponential prior for s_j takes the form of:

$$p(s_j|\lambda_j) \propto \frac{\lambda_j}{2} \; exp\left\{-\frac{s_j\lambda_j}{2}\right\} \quad [3.14]$$

In this chapter , we will use another formula of the prior Laplace distribution which is Scale Mixture Uniform (SMU) in Tobit quantile regression model for implementing the coefficients estimation and variables selection. Where, the Laplace prior of β_j coefficients in equation [3.14] is written as follows:

$$\frac{\lambda}{2} e^{\{-\lambda|\beta_j|\}} = \int_{s_j>|\beta_j|} \frac{1}{2u_j} \frac{\lambda^2}{\Gamma(2)} \; {u_j}^{2-1} \, exp\{-\lambda u_j\} \, du_j \quad ,$$

$$= \int_{|\beta_j|} \frac{1}{2u_j} \frac{\lambda^2}{\Gamma(2)} \; {u_j}^{2-1} \, exp\{-\lambda u_j\} \, du_j \quad [3.15]$$

Also equation [3.15] can be rewritten as a function from two parts. The first part is assigned to prior uniform distribution for u_j**,** and the second part belongs to a Gamma distribution with shape parameter (2) and scale parameter (λ), where λ has Gamma prior with parameters (a, b). This parameter is necessary for coefficient shrinkage. The prior distribution of α_θ is assigned to standard uniform prior . (a, b), are fixed hyperparameters are take initial values.
From the above information , our Bayesian hierarchical Tobit quantile regression model can be summarized as follows:

$$y_{i=max\{0,T_i^*\}}, \qquad i=1,\dots,n,,$$

$$T_i^*|\alpha_\theta, \beta_\theta, z_i \sim N(\alpha_\theta + x_i^T\beta_\theta + (1-2\theta)m_i, 2m_i),$$

$$p(\alpha_\theta) \propto 1,$$

$$m_i \sim Exp\,(\theta(1-\theta)), \qquad [1]$$

$$\beta_j|u_j \sim Uniform\,(-u_j, u_j),$$

$$u_j|\lambda \sim {u_j}^{2-1}\exp(-\lambda u_j),$$

$$\lambda \sim Gamma(a, b),$$

3.2.3: The Conditional Posterior Distributions Inference

The condition posterior distribution is produced from multiplying equation [3.12] and the set of equation [3.16] . It is a the probability distribution for each parameters of model. In this chapter constructing the Gibbs sampler depends on the following:
The conditional posterior distribution of variable (T_i^*) is distributed truncated normal distribution which is given by:

$$T_i^*|y_i, x_i, m_i, \alpha_\theta, \beta_\theta \sim \begin{cases} \gamma(y_i) & if\ y_i > o \\ N(\alpha_\theta + x_i^T\beta_\theta + (1-2\theta)m_i, 2m_i)I(T_i^* \le 0) & \text{otherwise} \end{cases} \qquad [1.20]$$

where $\gamma(y_i)$ is degenerate distribution. The full conditional posterior distribution of α_θ is normal distribution with mean equal $(\sum_{i=1}^n \frac{(T_i^*-x_i^T\beta_\theta-(1-2\theta)m_i)}{2m_i})$ and variance equal $(\sum_{i=1}^n \left(\frac{1}{2m_i}\right))$. Also, the full conditional posterior distribution of m_i is inverse Gaussian with mean $\sqrt{\frac{\mathbf{1}}{(T_i^*-\alpha_\theta-x_i^T\beta_\theta)^{\mathbf{2}}}}$ and shape parameter $(\frac{\mathbf{1}}{\mathbf{2}})$. And the full conditional posterior distribution of β_j is truncated normal with mean $\acute{\sigma}^2\sum_1^n \frac{x_{ij}\left(T_i^*-(1-2\theta)m_i-\sum_{j=1,j\neq k}^p x_{ij}\beta_j\right)}{2m_i}\ \mathrm{I}\left[|\beta_j| < u_j\right]$ and variance $\acute{\sigma}^2 = \left(\sum_1^n \frac{{x_{ij}}^2}{2z_i}\right)^{-1}$. The full conditional posterior distribution of u_j is left-truncated exponential with rate parameter(λ). The full conditional posterior distribution of λ is a Gamma distribution with rate parameter $(a + 2p)$ and scale parameter$\left(b + \sum_{j=1}^p|\beta_j|\right)$.

Where a,b are hyperparameters which are take initial values. Our Bayesian hierarchical posteriors will generate an attractive MCMC algorithm for our proposed method new Bayesian Lasso Tobit quantile regression (New B L Tobit Q Reg).

3.3: Chapter Conclusions

In this chapter, we propose a new Bayesian Lasso Tobit quantile regression method for variable selection and coefficients estimation in Tobit Q Reg model through assigning a prior independent scale-mixture of uniform (SMU) distributions for the regression coefficients. Then, a simple and efficient MCMC algorithm has been presented. Simulation studies and a real data set are used to investigate the performance of the proposed method compared to some other existing methods. Both simulated and real data examples show that the proposed method performs quite well compared to the other methods at a variety of scenarios.

Chapter Four

Bayesian Composite Tobit Quantile Regression

4.1: Introduction

Since the seminal work of (Powell (1986))[59], Tobit QReg (Tobit QReg) has received considerable attention in recent literature as well as numerous practical applications in a number of fields such as Econometrics, biological sciences, finance and medicine. A large body of work exists on classical estimation for Tobit QReg and we refer to (Hahn (1995))[33], (Buchinsky and Hahn (1998))[8], (Bilias *et al.* (2000))[9] and (Chernozhukov and Hansen (2008))[17] for a comprehensive review. The Tobit Q Reg model offers an active way of coping with left-censored data, and can be viewed as a linear Q Reg model where only the data on the response variable is incompletely observed. One of the attractions of Tobit QReg over its standard Tobit regression counterpart lies in its flexibility in providing a more complete description of the functional changes than focusing solely on the center of the distribution. Given the linear latent variable model,

$$T_i^* = \alpha + x^T\beta + \varepsilon_i, \qquad i = 1, \dots\dots, n \qquad [4.1]$$

where α is the intercept ,$x_i = (x_1, \dots\dots. x_k)^T, \beta = (\beta_1, \dots \dots \beta_k)^T$ and ε_i are independent with their θ_{th} quantiles to 0 and distribution function F. Powell noted that if we observe $y_i = max\{c, T_i^*\}$, where *c* is a known censoring point, then the conditional quantile functions,

$$Q_{y_{i|x_i}}(\theta|x_i) = \alpha_\theta + x_i^T\beta_\theta \qquad [4.2]$$

can be estimated consistently by the solution to the following minimization problem,

$$\min_{\alpha_\theta, \beta_\theta} \sum_{i=1}^{n} \rho_\theta\,(y_i - max\{c, \alpha_\theta + x_i^T\beta_\theta\}) \qquad [4.3]$$

where $\rho_\theta(t) = t(\theta - 1_{t\le0})$ is so called the check function of (Koenker and Bassett (1978))[38] at a quantile $\theta\epsilon(0,1)$ and 1 is the indicator function. It is well known that the asymptotic theory for Q Reg models have been well developed (Koenker, ((2005)[42]. However, inference for these models is difficult, especially for censored data (Reich et al., (2010))[61]. In contrast, a Bayesian framework enables exact inference, even when *n* is small and is well suited to incorporate censored data. Because the check function (4.3) is closely related to the skewed Laplace distribution (SLD) (Koenker and Machado, (1999))[48]; (Yu and Moyeed, (2001))[], (Yu and Stander (2007))[] proposed a Bayesian method for analyzing QReg model for censored data using the SLD for the errors. (Ji *et al* (2012))[37] studied the problem of variable selection in Tobit Q Reg via Gibbs sampler based on the scale-mixture expression in (Kozumi and Kobayashi (2011))[49]. (Kobayashi and Kozumi (2012))[51] considered Bayesian analysis of QReg for censored dynamic panel data. (Alhamzawi and Yu (2015))[] proposed a Bayesian approach for variable selection and coefficient estimation in TQReg model using g-prior distribution with ridge parameter and (Alhamzawi (2014))[3] proposed a Bayesian approach for Tobit Q Reg with the elastic net penalty. The aforementioned methods for modeling TQReg focus on a single quantile level. However, for a given distribution, the efficiency of TQReg estimators depends on the quantile level. Because the distribution is unknown, it is difficult to select the most informative quantile which can provide an efficient estimator.

In the context of quantile regression, (Zou and Yuan (2008))[76] showed that when the errors are independently and identically distributed, composite Q Reg (regression at multiple quantiles) offers a more efficient estimator than the estimator obtained using Q Reg at a single quantile level. The authors showed that the relative efficiency of composite Q Reg compared to the OLS is greater than 70% regardless the distribution of residuals. (Bradic *et al.* (2011))[12] proposed a robust and efficient penalized composite quasi-likelihood method for ultrahigh dimensional variable selection. (Kai *et al.* (2010))[47] proposed a local linear composite estimator for estimating the nonparametric regression function, and (Zhao and Xiao (2014))[77] considered a weighted composite QRegand proved its oracle properties. (Huang and Chen (2015))[34] proposed a Bayesian formulation of composite QRegusing the asymmetric Laplace distribution for the errors and sampling the regression coefficients from its posterior using a Gibbs sampler . In the current chapter, we propose a Bayesian approach for composite Tobit QReg (Composite Tobit QReg). The approach is illustrated via simulation studies and a real data set. Results show that combine information across different quantiles can provide a useful method in efficient statistical estimation. Based on the simulation studies and real data analysis, we argue that it is necessary to combine quantile information based on estimators at different quantiles to achieve efficiency gain.

4.2: The proposed method

4.2.1: Bayesian formulation of the CompositeTobit QReg

Consider H different quantiles ,$0 < \theta_1 < \theta_2 < \cdots < \theta_H < 1$ Let $y_i = \alpha + x_i^T \beta + \varepsilon_i$ for $i = 1, \ldots\ldots, n$ and $h = 1, \ldots\ldots, H$,. Then, the composite QReg(Zou and Yuan,(2008))[76] is given by

$$\left(\hat{\alpha}1, \hat{\alpha}2, \ldots, \hat{\alpha}\text{H}, \hat{\beta}\right) = \underset{\alpha_1 \ldots\ldots \alpha_2, \beta}{Min} \quad \sum_{h=1}^{H} \left\{ \sum_{i=1}^{n} \rho_{\theta_h} \left(y_i - \alpha_h + x_i^T \beta\right) \right\}, \qquad [4.4]$$

Where $\rho_{\theta_h}(t) = t(\theta_h - 1_{t \le 0}), \theta_h = \frac{h}{H+1} for\ h = 1, \ldots., H.$

Next, assume that $T_i^* = \alpha + x^T \beta + \varepsilon_i$ and $y_i = max\{c, T_i^*\}$. We propose composite Tobit QReg which solves the following:

$$\left(\hat{\alpha}1, \hat{\alpha}2, \ldots, \hat{\alpha}\text{H}, \hat{\beta}\right) = \underset{\alpha_1 \ldots\ldots \alpha_2, \beta}{Min} \quad \sum_{h=1}^{H} \left\{ \sum_{i=1}^{n} \rho_{\theta_h} \left(y_i - \max(c, \alpha_h + x_i^T \beta\right) \right\}, \qquad [4.5]$$

Although Equation (4.5) is not differentiable at 0, the minimization can be achieved through some modifications to algorithm proposed by (Koenker and D'Orey (1987))[46]. However, this algorithm might be inefficient at lower quantiles of left censored data. In this paper, to estimate the Composite Tobit QReg parameters, a Bayesian method is considered and a new Gibbs sampler is proposed for posterior inference.

Now, if we assume ε_i being independent and identically distributed random variables from the SLD where the density function of the SLD with a scale parameter of 1 is

$$f(\varepsilon|\theta_m) = \theta_m(1 - \theta_m) exp\{-\rho_{\theta_m}(\varepsilon)\}. \qquad [4.6]$$

Then the joint distribution of $y = (y_1, \ldots, y_n)^T$ $given$ $X = (x_1, \ldots, x_n)^T$,$\alpha = (\alpha_1, \ldots . \alpha_h)^T$ and $\beta = (\beta_1, \ldots . \beta_k)^T$ for composite Tobit QReg is

$$f(y|X, \alpha, \beta) = \prod_{h=1}^{H} \theta_h^n (1 - \theta_h)^n exp \left\{ - \sum_{i=1}^{n} \rho_{\theta_h} (y_i - max\{c, \alpha_h + x_i^T \beta\} \right\} \qquad [4.7]$$

Hence, minimizing Equation (4.5) is equivalent to maximizing the likelihood function of the censored response y_i (4.7). It is worth pointing out that assuming SLD for y_i is merely an

articial assumption used to achieve the possible parametric connection between the minimization in Equation (4.5) and the maximum likelihood theory (Yu et al., (2013))[]; (Benoit et al.,(2013))[13]. It is very difficult to solve Equation (5) directly because of the mixture of H components. Following (Huang and Chen (2015))[34],
we use a cluster assignment matrix C whose $(i,h)^{th}$element C_{ih} is equal to 1 if the i_{it} subject belongs to the h_{th} cluster, otherwise $C_{ih} = 0$. The element C_{ih} is treated as missing values. Thus, our likelihood takes the form
of

$$f(y|X,\alpha,\beta) = \prod_{h=1}^{H}\prod_{i=1}^{n}\left[\theta_h(1-\theta_h)exp\{\rho_{\theta_h}(y_i - max\{c,\alpha_h + x_i^T\beta\})\}\right]^{C_{ih}} \qquad [4.8]$$

Recently, (Kozumi and Kobayashi (2011))[49] demonstrated that the distribution of the SLD can be reformulated as a mixture of normal distributions. For our Bayesian composite Tobit Q Reg, the formulation of (Kozumi and Kobayashi (2011))[49] can be written as: If
$y_i|\alpha_m,\beta,z_i \sim Normal\ (max\{c,\alpha_h + x_i^T\beta\} + (1-2\theta_h)m_i, 2m_i)$,
$m_i \sim Exp\big(\theta_h(1-\theta_h)\big)$ [4.9]

$$f(T_i^*|\alpha_h,\beta,z_i) = \prod_{i=1}^{n}\frac{1}{\sqrt{4\pi m_i}}\ e^{\frac{1}{2}\sum_{i=1}^{n}\sum_{h=1}^{H}\frac{c_{ih}\,(T_i^*-\alpha_h-x_i^T\beta-(1-2\theta_h)m_i)^2}{2z_i}}$$

Where m_i are independent ,then marginalizing over m_i gives us $y_i|\alpha_h,\beta \sim SLD(max\{c,\alpha_h + x_i^T\beta\},1,\theta_h$ .Under this formulation, the composite Tobit Q Reg coefficients has desirable conditional conjugacy features for building a tractable and efficient Gibbs sampler algorithm for fitting the model to the data.

4.2.2. Prior Specification

To proceed a Bayesian analysis, we specify a uniform prior distribution for α_m, $p(\alpha_m) \propto 1$. We assign a zero mean normal prior distribution for β_j taking the form of

$$p(\beta_j|s_j) = \frac{1}{\sqrt{2\pi s_j}}exp\left\{-\frac{\beta_j}{2s_j}\right\}, \qquad [4.10]$$

where s_j is the unknown prior variance of β_j .Then we assign an exponential prior on s_j takes the form of .

$$p(s_j|\lambda_j) \propto \frac{\lambda_j}{2}\ exp\left\{-\frac{s_j\lambda_j}{2}\right\}, \qquad [4.11]$$

where λ_j is unknown hyperparameter. Since (10) involves a zero mean Gaussian prior for the components of β with unknown variances, these components are shrunk. The degree of shrinkage is controlled by the prior variance $\lambda_j, j = 1, \dots, k$.. This two-level prior can provide adaptive regularization of weights for the composite Tobit QReg parameters β and represent an alternative model to the Bayesian Lasso model. We further put a gamma prior on λ_j as

$$p(\lambda_j|a,b) \propto \lambda_j^{a-1}exp\{-b\lambda_j\}, \qquad [4.12]$$

where $a\ and\ b$ are two fixed hyperparameter. Following (Li et al. (2010))[2010] and Hashem et al. (2015), we set a and b as small values ($a = 0.1$ and $b = 0.1$) so that the prior for λ_j is essentially non-informative . To summarize, our Bayesian hierarchical model given by
$y_{i=max\{c,T_i^*\}}, \quad i=1,\dots,n,$

$T_i^*|\alpha_h, \beta, z_i \sim [N(\alpha_h + x_i^T\beta + (1-2\theta_h)m_i, 2m_i)]^{c_{ih}}$,

$p(\alpha_h) \propto 1$ [4.13]

$m_i \sim Exp\ (\theta_h(1-\theta_h))$,

$\beta_{j\sim N(s_j)}$

$s_{j\sim} Exp\left(\frac{\lambda_j}{2}\right)$,

$\lambda_j \propto \lambda_j^{a-1} exp\{-b\lambda_j\}$.

4.2.3 Posterior Computation Inferences

Therefore the hierarchical model to composite Tobit Q Reg model with Lasso penalty are:

The full conditional posterior distribution of latent variable (T_i^*) is given by

$$T_i^*|y_i, m_i, \alpha_h, \beta \sim \begin{cases} \{\Upsilon(y_i), & if\ y_i > c; \\ \left\{\prod_{h=1}^{H} [N(\alpha_h + x_i^T\beta + (1-2\theta_h)m_i, 2m_i)]^{c_{ih}}\right\} I(T_i^* \leq c), otherwise \end{cases} \quad [1.28]$$

where c is equal zero and $\Upsilon(y_i)$ denoted to a degenerate distribution. The full conditional posterior distribution of m_i for $i = 1, \ldots, n$, is the Inverse Gaussian with mean $\sqrt{\sum_{h=1}^{H} C_{ih}/(T_i - \alpha_h - x_i^T\beta)^2)}$ and shape parameter $\sum_{h=1}^{H} C_{ih}/2$. The full conditional posterior distribution of α_h is normal distribution with mean $(\tilde{\sigma}_h^2 \sum_{i=1}^{n} C_{ih}\,(T_i^* - x_i^T\beta - (1-2\theta_h)m_i)/2m_i)$ and variance $(\sum_{i=1}^{n}(C_{ih}/2m_i))^{-1}$. The full conditional posterior distribution of β_j[j=1, 2,….k] is normal distribution with mean $(\tilde{\sigma}_j^2 \sum_{h=1}^{H}\sum_{i=1}^{n} C_{ih}x_{ij}(y_i - \alpha_h - \sum_{l\neq j} x_{il}\beta_l - (1-2\theta_h)m_i)/(2m_i))$ and variance $(\tilde{\sigma}_j^2 = (\sum_{i=1}^{n}\sum_{h=1}^{H} C_{ih}\, x_{ij}^2/2m_i) + s_j^{-1})^{-1})$. The full conditional posterior distribution of $s_i^{-1}[j = 1,2, \ldots \ldots k]$is the inverse Gaussian with mean $\sqrt{\lambda_j/\beta_j^2}$ and shape parameterλ_j. The full conditional posterior distribution of λ_j is gamma distribution with shape parameter $(a + 1)$ and scale parameter $(b + \frac{s_j}{2})$. The full conditional posterior distribution of $C_{ih} = (C_{i1,} C_{i2,} \ldots \ldots, C_{iH})^T$ is a multinomial distribution; $p(C_i|y, X, \alpha, \beta, z) \propto$ $\text{multinomial}(1, \hat{p}_1, \ldots., \hat{p}_H)$, where $\hat{p}_H = \frac{\exp[-(y_i - \alpha_h - x_i^T\beta - (1-2\theta_h)m_i/2m_i]}{\sum_{h=1}^{H}\exp[-(y_i - \alpha_h - x_i^T\beta - (1-2\theta_h)m_i/2m_i]}$. From the full conditional posterior distributions are show in above formulas, we will obtain a simple and efficient Gibbs sampler algorithm. Our algorithm was run for 16,000 iterations and the first 1000 were removed as burn in. Then, we think the subsequent iterations by keeping every 5th simulation draw and discarding the rest.

4.3: Chapter Conclusion

Composite Q Reg models have been shown to be effective techniques in improving the prediction accuracy (Zou and Yuan, (2008))[76]; (Bradic et al.,(2011))[12]; (Zhao and Xiao, 2014))[77]. In this chapter, we developed a simple and efficient MCMC based computation technique for composite Tobit quintile regression model based on a mixture of an exponential and a scaled normal distribution of the skewed Laplace distribution. Simulation studies show that our proposed method is effective in coefficient estimation under different distributions. Based on the simulation studies and real data analysis, we argue that it is necessary to combine quantile information based on estimates at different quantiles to achieve efficiency gain.

Chapter Five

Analysis of Factors Affecting Iraqi Banks' Investments by Using a New Bayesian Lasso Tobit Q Reg Model and Bayesian Composite Tobit Q Reg model

5-1 : Introduction

Banking investments are considered one of the financial resources for achieving high profit in the future. Some banks invest part of their funds in various economic projects via diverse banking investments, while some banks do not have any activity in banking investments. Banking investments are one of the most important banking advantages (Fohlin, C: (2014))[22]; (White, E. N: (1986))[70]. All banks are seeking to reach the highest possible return of investments under reduced risks. Banks are looking to invest by purchasing securities with the highest return and the lowest risk. The investments of the Iraqi banks were of 4.8 trillion Iraqi dinars at the end of 2013. These banking investments are distributed into two parts: the first part represents investments in Iraqi markets reaching 3.1 trillion Iraqi dinars, and the second part represents investments in foreign markets reaching 1.7 trillion Iraqi dinars. The state-owned banks invested 1.9 trillion Iraqi dinars in Iraqi markets and 1.4 trillion Iraqi dinars in foreign markets. Private banks invested 1.2 trillion Iraqi dinars in Iraqi markets and 254.5 billion Iraqi dinars in foreign markets. The total of banking investments in Iraqi and foreign markets were 4.8 trillion Iraqi dinars which contributed with 1.8% to the gross domestic product (Iraq Central Bank (2013))[36].

The present work aims to analyze the influence of certain variables on the Iraqi banks' investments. These variables are: banking deposits, banking profits, bank capital, bank reserves, banking loans, advertising expenses, age of the bank, number of bank branches, bad debt. These variables have impact and relative importance in Iraqi banks' investments. In the current study, the response variable (Iraqi banks' investments) is censored response variable from left side at zero. Therefore, the Tobit regression model (Tobit Reg model) is considered more adaptable with censored response data. Nevertheless, our data have a set of problems, such as: (a) the distribution of banking investments data are skewed to right side; (b) the banking investments data have a big gap between the smallest value and the largest value - therefore, our data contain much outlier values, etc. Due to these problems, the Tobit Reg model is no longer appropriate with our data. In order to overcome these problems, the Tobit quantile regression (Tobit Q Reg) model has been used. Additionally, Tobit Q Reg gives us a complete information of the relationship between Iraqi banks investments and a set of independent variables. In this thesis, we used a new Bayesian Lasso in Tobit quantile regression model for describing the relationship between censored response variable (Iraqi bank investments) and a set of independent variables, and also to identify the relative importance for these independent variables in studied models via thirty Tobit quantile levels. Also we use the Bayesian composite Tobit quantile regression model to assess the relationship between the censored response variable (Iraqi bank investments) and a set of independent variables, and also to identify the relative importance for these independent variables in composite Tobit Q Reg via six groups of Tobit quantile levels.

5.2- Brief Explanation about all Banks Active in Iraq

In this part, we detail the number and types of banks active in Iraq. This sector has witnessed a growth in number of banks which increased from 10 to 47 at the end of 2013. The banks active on the Iraqi market are divided into three types, as follows:

5.2.1: State-Owned Banks:

There are 47 banks active in Iraq including 7 state-owned banks. These banks are Al-Rashid Bank, Rafidain Bank, Commercial Bank of Iraq, Industrial Bank of Iraq, Agricultural Cooperation Bank, Real Estate Bank and the Iraqi Bank.

5.2. 2: Private Banks

Starting with the first of October 2006, the number of private banks active in Iraq has become 25 banks. All these banks have a license from the Iraqi Central Bank. The number of Iraqi private banks were 18 banks until late 2003. They began their activity in the early 1990s. At the end of December 2013, the number of private banks active in Iraq increased to thirty (Dunia report (2014))[18].

5.2. 3: Foreign Banks

A foreign banks are obligated to implement the laws and regulations of both the homeland state and the host state (Nasr, S et al. (2011))[57]. The foreign banks provide to Iraqi customers a set of banking service such as, banking loans and banking deposits etc. At the end of 2013, the number of a foreign banks reached ten banks

5.3: Study Sample and Mathematical Model

The data were extracted from the report of the Central Bank of Iraq (2013). The sample size is 47 observations (the number of banks active in Iraq). In the current study, the sample contains one response variable which is the Iraqi banks' investments. It can either take positive quantities, when the banks have investments activity or can be zero, when these banks haven't any investment activity. This means, that the response variable (Iraqi banks' investments) will be left censored at (zero). The banks' investments are affected by a set of variables, directly or indirectly. To evaluate the relationship between the response variable and a set of independent variables statistical methods are employed. In the current study, the response variable is being censored at zero point. Therefore, the more appropriate model is a Tobit regression model. But to obtaining the entire coverage of the full relationship distribution between the response variable (Iraqi banks' investments) and a set of independent variables we will use the Tobit quantile regression model at a specific Tobit quantile levels (determined by researcher). Also, we will use a composite Tobit quantile regression model. Therefore, the mathematical model for the current study takes the following formula:

$$y_i = \max(0, T_i^*), T_i^* = \alpha + \beta_{1\theta}x_{1j} + \beta_{2\theta}x_{2j} + \beta_{3\theta}x_{3j} + \beta_{4\theta}x_{4j} + \beta_{5\theta}x_{5j} + \beta_{6\theta}x_{6j} + \beta_{7\theta}x_{7j} + \beta_{8\theta}x_{8j} + \beta_{9\theta}x_{9j} + u_{i\theta} \qquad j=1,2,\ldots\ldots 47$$

where θ is a value belonging to the open interval (0,1). So, there are an infinite of Tobit quantile levels. In the current study, we assess the relationship between censored response variable and a set of independent variables by two methods: new Bayesian Lasso in Tobit Q Reg at thirty Tobit quantile levels and Bayesian composite Tobit Q Reg at six groups of Tobit quantile levels. The assessment is done in two steps: first the coefficients estimation of the models by two methods, and second, the computation of relative importance to independent variables also by two methods.

5.3.1: The Independent Variables

5.3.1.1. x_1 Banking Deposits

A deposit is the amount of money placed by the owner into a banking institution, representing the liability owed by the bank to the customer. Usually, the amounts can be kept in current accounts, savings accounts, time deposit accounts, call deposits or any other form of account that allows the customer access to the funds. The bank uses the deposited amounts to finance the banking loans it grants to customers.

5.3.1. 2. x_2 Banking Profits

Financial institutions carry a key role within the financial sector in the economy of countries. Thus, the banking system maintains a wide range of financial products and services to potential customers. Functioning financial institutions imply that banks obtain profits from running their operations. Mainly, a bank can profit in case the expenses are lower than the earnings. The earnings of banks originate from fees, commissions and interest paid by customers from using the services of the bank (its assets). Such assets are banking loans granted to private individuals or businesses. In return, the bank pays interest on liabilities, such as the bank's deposits or banking loans from other banks.

5.3.1. 3. x_3 Banking Capital

The capital of the bank is the difference between a bank's assets and its liabilities. Higher capital indicates that the bank can survive unexpected losses, and following the 2008 financial crisis, the capital of the banks has been followed more closely. The crises revealed some regulation flaws regarding banks' capital, which have since then been included in the Basel III accord. Basel III aims to increase stability and transparency of the banks. The main change is tightening capital requirement for banks by increasing liquidity. Capital is supposed to have a negative impact on profitability – the higher the capital of a bank is, the lower the profitability. However, the work of Osborne, Fuertes and Milne demonstrates that the relationship between capital and profitability is fluctuating according to market conditions (Hala Hijazi, (2017))[35]. Therefore, the size of the banking capital depends on the economic situation of the country. After 2003, Iraq has recovered economically, so Iraqi banks must increase their banking capital, for meeting the requirements of activity in an economic environment.

5.3.1. 4. x_4 Bank Reserves

Banking reserves are amounts deposited with the central bank or internally by the banks in their vaults. Minimum reserves requirements levels are established by central banks based on the balance sheet of the bank. The funds need to be deposited for a certain period of time, called the maintenance period and at the end of the maintenance period, the central bank pays to banks the interest for the amount deposited.

5.3.1. 5. x_5 Banking Loans

A loan is a debt provided by the bank to the borrower based on a contract stating the loan conditions – the principal of the loan (the amount lent to the borrower), the interest rate paid by the borrower, date of repayment. From the banks' point of view, banking loans bring the most profit (the interest rate paid by the borrower). They also imply costs primarily there are the cost of funds at the time the loan is made, loan administration costs (loan analysis and execution fees), risk costs. However, banks tend to reduce the charges of a loan due to stronger competition or to underestimated risks related to banking loans.

5.3.1. 6. x_6 Advertising Expenditures

Some banks depend on advertising for attracting customers, especially newer banks. Some older banks support various television programs as social responsibility (Ors, E. (2006))[58]. This variable is measured by the amount of advertisement expenses,

5.3.1. 7. x_7 Age of The Bank

The oldest surviving bank was established in Europe, in 1472. This proves the continuity of financing institutions over a very long period of times. While in the modern world, the distinction between old banks and new banks is not significant, there are psychological reasons that may influence customers as well as partners to choose a bank over another based on their age – an older bank may seem more experienced and trustworthy for customers with lower innovation needs and expectations (Ikechukwu, I. O et al., (2016))[19], while newer banks may seem more flexible and more willing to incorporate new technologies into their products and services. Measuring this variable depends on the number of years since the beginning of banking activity until 31st December 2013.

5.3.1.8. x_8 Number of Bank Branches

The number of bank branches has significant impact in various aspects of the bank's activity. As examined in the work of De (Haan and Poghosyan, (2012))[] one of the main aspects identified is the bank's products and services offer large banks tend to have more diversified offer than smaller banks. Also, larger banks are more risk-inclined, as their size prevents them from failing

5.3.1. 9. x_9 : Bad Debt

Bad debt represents the amount owed by the customer to the bank which is unlikely to be paid back or for which the collecting costs are higher than the actual amount owed. Bad debt results mainly from nonperforming banking loans (NPL). The definition of a nonperforming loan can vary, but it is mainly a loan for which the debtor has not made any payments for a certain period. After this period, the possibility of the loan being repaid is very low.

5.4 - New Bayesian Lasso in Tobit Q Reg:

We will use our proposed methods of New Bayesian Lasso Tobit Q Reg in two directions: the coefficients estimation and variable selection via thirty Tobit quantile levels.

5.4.1 Coefficients Estimation by The New Bayesian Lasso Tobit Q Reg

This method is used to evaluate the relationship between Iraqi banks investments and a set of independent variables via thirty Tobit quantile regression lines.

5.4. 2 Variable Selection by New Bayesian Lasso Tobit Q Reg

The our method is used to identify the active independent variables in "Iraqi banks investments" depending on the relative importance of these variables, via thirty different Tobit quantile levels.

5.5: Bayesian Composite Tobit Quantile Regression:

We will use our proposed methods of Bayesian composite Tobit Q Reg in two directions: the coefficients estimation and variable selection via six groups of Tobit quantile levels.

5.5.1: Coefficients Estimation of Composite Tobit Q Reg Model

In our models, we will use the Bayesian approach for estimating the models coefficients because the Bayesian approach has very suitable features. In this section, we will employ it for coefficients estimation for six composite Tobit Q Reg models according to six groups of Tobit quantile levels (H=5, H=10, H=15, H=20, H=25 and H=30):

5.5.2. Variable Selection of Composites Tobit Q Reg Model

we will identify the relative importance of informative independent variables in the composite Tobit Q Reg models through six groups of Tobit quantile levels .

Chapter Six

SUMMARY CONCLUSIONS AND FUTURE RESEARCHES

The main objective of this thesis is to implementing of Bayesian variable selection and coefficients estimation in Tobit quantile regression model via a set of a new proposed methods. The our proposed methods in Tobit Q Reg model and composite Tobit Q Reg model are consider a new addition in regularized Bayesian approach. And these our proposed methods are very efficient compared with other methods in same field, this clear from results of simulation examples and real data which were used. The our proposed methods (new Bayesian Lasso Tobit Q Reg, and Bayesian composite Tobit Q Reg) are used to analyse the Iraqi banks' investments data. Where, our method (new Bayesian Lasso Tobit Q Reg) is used in two sides: firstly, it is used to modelling the relationship between Iraqi bank investments and a nine independent variables at thirty Tobit quantile levels. Secondly, it is used for determining relative importance of independent variables in Tobit Q Reg model also at thirty Tobit quantile levels to achieving variable selection. Our method Bayesian composite Tobit Q Reg is used in two sides firstly, it is used to modelling the relationship between Iraqi bank investments and a nine independent variables at six groups of composite Tobit quantile levels. Second, it is used for determining of relative importance to independent variables in composite Tobit Q Reg model also at six groups of composite Tobit quantile levels to achieving variable selection. From study of theoretical and Applied to our proposed methods , we will obtained the following conclusions.

6.1 .THEORETICAL CONCLUSIONS:

From the results of the simulation and real data studies to our proposed methods, we will arrive to the following conclusions:

- In our proposed method, new Bayesian Lasso quantile regression (new Bayesian Lasso Q Reg) is assigned independent scale-mixture of uniform distributions for the regression coefficients. Then, a simple and efficient MCMC algorithm was presented for Bayesian sampler. Simulation studies and a real data set are used to investigate the performance of the proposed method compared to some other existing methods. Both simulated and real data examples show that the proposed method performs quite well compared to the other methods under a variety of scenarios.
- Simulation studies show that our Gibbs sampler is effective in shrinkage and estimation of the regression coefficients under a variety of scenarios. Also our simulation scenarios indicate that our proposed method works well even when the true distribution for the error term is not asymmetric Laplace distribution (ALD). This case was also observed by

Yuan and Yin (2010))[71], (Li et al. (2010))[52], (Alhamzawi et al. (2012))[1] and (Ji et al. (2012))[37], among others.

- The proposed method (new Bayesian quantile regression) gives us an efficient and simple Gibbs sampling algorithm with good conditional posterior distributions. The proposed Gibbs sampling algorithm has many advantages ans it not complicated to use, since it implementing variables selection is obvious and our method is more balanced compared with other methods under study.
- Our proposed MCMC algorithm presented for the new Bayesian Lasso Q Reg method was very stable, as it is shown in the results which belong to multivariate potential scale reduction factor (MPSRF). Our method has become stable and close to one after 2000 iterations via each quantile levels (0.50, 0.75 and 0.95), showing that the convergence of the posterior distribution for the proposed method was rapid and the mixing was good.
- Our proposed method (new Bayesian Lasso Q Reg) considers a new addition in field of Bayesian regularized Q Reg model. Through reformulation, the Laplace prior distribution becomes new formula of mixed distribution between uniform distribution and Gamma distribution, (scale mixture uniform).

- Our results of the simulation and real data approach indicate that the proposed method (new Bayesian Lasso Q Reg) has a good performance compared with other non-Bayesian and Bayesian methods. In addition, our proposed method is to consider one of the competitive method in coefficients estimation variable selection, and forecast accuracy.
- One of the good and the valued features of Q Reg model has a robustness and it works well even if it violates the normal supposition of error etc. But, in parametric Bayesian Q Reg framework, we assumed the error belongs to asymmetric Laplace distribution (ALD). While this assumption causes some worry on loss (check) of the nature of nonparametric Q Reg model. So, the results of our proposed method (new Bayesian Lasso Q Reg) is quite insensitive to this assumption and behaves good for data generated from other error distributions.
- To assess our proposed method (new Bayesian Lasso QReg), we tested it with three other methods, first method belongs to non-Bayseian methods and second and third methods belong to Bayesian methods. From results which belong to direct way criterion, the coefficients estimation belonging to our proposed method is much closed with true parameters compared with non-Bayesian and Bayesian methods. This indicates that our proposed method was better than other methods.
- The entire literature within the field of regularized Bayesian Q Rag models used transformation Laplace distribution to scale mixture of normal distribution which was proposed by (Andrews and Mallows: (1974)) [5]as the following formula:

$$\frac{\lambda_j}{2}\,e^{-\lambda_j|\beta_j|} = \int_0^\infty \frac{1}{\sqrt{2\pi s_j}} e^{\left(-\frac{\beta_j^2}{2s_j}\right)} \frac{\lambda_j^2}{2} e^{\left(-\frac{s_j\lambda_j^2}{2}\right)} ds_j$$

In order to obtain efficient and simple algorithms for coefficients estimation and variable selection in Q Reg model see (Li et al. (2010))[2010], (Alhamzawi et al. (2012)) [1]among others . In our proposed method (new Bayesian Lasso Q Reg) another transformation form for Laplace distribution was used, that is scale mixture uniform as the following formula:

$$\frac{\lambda}{2} e^{\{-\lambda|\beta_j|\}} = \int\limits_{u_j>|\beta_j|}^{\infty} \frac{1}{2u_j} \frac{\lambda^2}{\Gamma(2)}\; {u_j}^{2-1}\, exp\{-\lambda u_j\}\, du_j$$

This formula was proposed by (Mallick and Yi (2014))[55]. From the new structure of Laplace distribution, we obtained a simple and tractable algorithm of our proposed method for the variables selection and coefficients estimation in Q Reg model.

- The feature of the proposed method (New Bayesian Lasso Q Reg) is that the proper prior distribution is flexible with different quantile levels. The behaviour of mixture of uniform prior distribution is obviously perfectly robust with several quantile levels. The mixture of uniform prior distribution is very important in regularized Bayesian Q Reg model. Therefore, our method of New Bayesian Lasso QReg seems to be very important in numerous applications, for instance the variable selection and longitudinal studies.
- Our proposed method has a good performance with real data. For this purpose air pollution data has been used. This dataset was measured by the Public Roads Administration in Norway and it consists of 500 observations, 7 covariates and one response variable. The mean square error is generated by our proposed method is smaller than the mean square error which is generated by other methods. In one case, we see in the real data study that the non-Bayesian method (rq) was the best from Bayesian methods, until from our proposed method (new Bayesian Lasso QReg) at middle quantile level ($\theta_2 = 0.75$). But our proposed method has recorded a good performance compared with other methods via a majority of quantile levels.
- In this thesis we used extensions from the new Bayesian Lasso Q Reg method to the new method as new Bayesian Lasso Tobit Q Reg is considered a new method within the regularized Bayesian Tobit Q Reg model, achieving variables selection and coefficients estimation together. The applied Bayesian hierarchy for generating full conditional posterior distributions is calculated from the joint conditional density function and scale mixture uniform prior distribution to create attractive Gibbs sampling algorithm.
- Our proposed method is to generate a new Bayesian hierarchy Lasso by using scale mixture uniform (SMU) prior distribution to coefficients of Tobit Q Reg model, in order to achieve coefficients estimation and variables selection, where, SMU is considered a good replacement for scale mixture normal (SMN) to regularized Bayesian Lasso Tobit Q Reg model. Our MCMC algorithm derived from full conditional posterior distributions, simple and efficient. The performance of our proposed assessed method compared with other methods by simulation examples and real data. The results in both simulation study and real data recorded our proposed method as new Bayesian Lasso Tobit Q Reg better than compared with other methods via different quantile levels. Therefore, it can be considered a good method for coefficients estimation and variable selection in Tobit Q Reg model.
- When the model accommodates a big number of independent variables, there is no guarantee that the penalty parameter for model complexity is suitable for achieving variables selection with many dimensions. Therefore our proposed method (new Bayesian Lasso Tobit Q Reg) considers a good approach for achieving variable selection in Tobit Q Reg model with many dimensions. Also, our proposed method does not need long time for achieving variables selection which is done automatically.
- Our propose method (new Bayesian Lasso in Tobit Q Reg) uses scale mixture of the uniform distribution (SMU) for a new hierarchy prior distribution to the model parameters which are lead to production of full conditional posterior distributions to constructing tractable and efficient (MCMC) algorithm for implementing of the variables selection and coefficients estimation in Tobit Q Reg model.

Also all authors that work within the field regularized Bayesian Tobit Q Reg models use transformation Laplace distribution to scale mixture of normal distribution, proposed by (Andrews and Mallows: (1974))[5] as the following formula.

$$\frac{\lambda_j}{2}\, e^{-\lambda_j|\beta_j|} = \int_0^{\infty} \frac{1}{\sqrt{2\pi s_j}} e^{\left(-\frac{\beta_j^2}{2s_j}\right)} \frac{\lambda_j^2}{2} e^{\left(-\frac{s_j\lambda_j^2}{2}\right)} ds_j$$

.

For obtaining efficient and simple algorithm for coefficients estimation and variable selection in Tobit Q Reg model (see(Ji et al. (2012))[37], Alhamzawi (2014))[3] among others.

In our proposed method (new Bayesian Lasso Tobit Q Reg) is used another transformation form for Laplace distribution is scale mixture uniform as the following

$$\frac{\lambda}{2} e^{\{-\lambda|\beta_j|\}} = \int_{s_j>|\beta_j|}^{\infty} \frac{1}{2u_j} \frac{\lambda^2}{\Gamma(2)}\; {u_j}^{2-1}\, exp\{-\lambda u_j\}\, du_j$$

The formula was proposed by (Mallick and Yi (2014))[55]. From the new structure of Laplace distribution, we obtained on straightforward and tractable algorithm for variables selection and coefficients estimation in Tobit Q Reg model.

- Our Gibbs sampling algorithm generates a complete and informative conditional posterior distribution in variables selection in Tobit Q Reg model. These advantages in our Gibbs sampling belong to using proper prior distribution, which is compound of two parts. The first part is assigned to prior uniform distribution, and the second part is assigned to Gamma distribution with shape parameter two and scale parameter. This proper hierarchy prior distribution provides to our Gibbs sampling algorithm durability with high dimensional models compared with other methods which are lost this advantages.

- The coefficients estimation in Tobit Q Reg model is implemented by minimization the following check (loss) function

$$\underset{\alpha_\theta,\beta_\theta}{min} = \sum_{i=1}^{n} \rho_\theta\, (y_i - max\{0, T_i^*\})$$

However, it is not differentiable at origin point, so there is not an exact form of the solution for these parameters (Koenker, (2005))[42]. The minimization of this check function can be resolved by a linear programming algorithm (Koenker and D'Orey, (1987))[46]. Therefore, our proposed method considers a new method which will contribute in coefficients estimation in Tobit Q Reg model and it considers efficient methods when the response variable has high censored data.

- For assessing our proposed method (new Bayesian Lasso Tobit quantile regression), we compared it with two other methods, the first one belongs to non-Bayseian methods and the second method belongs to Bayesian methods. Results are generated via four simulation studies, which are concluded with the behaviour of our proposed method in outperformance on Bayesian and non-Bayesian method until censored data. Also the coefficients estimation which belongs to our proposed method is closed with true parameters compared with non-Bayesian and Bayesian methods. This indicates outperforms of our proposed method compared with other methods.

- Our proposed method considers a good approach with real data. In order to evaluate our proposed method, the extramarital Affairs data has been used. It is introduced by Fair in (1978). This data are was found in AER package from R. The mean square error generated by our proposed method is smaller than the mean square error which is generated by the other methods. Therefore, our proposed method has a better performance than other methods with real data.

- Our MCMC algorithm belongs to our method (new Bayesian lass Tobit Q Reg), which is strong and attractive to achieve variable selection via computation of the relative importance to each covariates in our l Tobit Q Reg model.

- Composite Q Reg models have been shown to be influential techniques in developing the prediction accuracy (Zou and Yuan, (2008))[76]; (Bradic et al., (2011))[12]; (Zhao and Xiao, 2014)[77]. Our proposed method studies composite Tobit Q Reg from a Bayesian approach. An efficient and simple MCMC-based calculation method is derived for posterior distribution inference using a mixture of an exponential and a scaled normal distribution of the asymmetric Laplace distribution (ALD). The approach is studied via simulation examples and a real data. These results show that gathered information across different quantile levels can provide a good method in efficient statistical estimation. This is considered first work to study composite Tobit quantile regression by a Bayesian perspective.

- In Tobit quantile regression model, there are an infinity numbers of Tobit Q Reg lines at different Tobit quantile levels. Therefore, the process of choosing the best Tobit Q Reg line is hard matter. To overcome this problem it is necessary to use composite Tobit quantile regression in order to obtain estimators at different quantile levels to achieve efficiency gain. Our proposed method (Bayesian Composite Tobit Quantile Regression) considers a new addition in coefficients estimation of composite Tobit Q Reg model.

- In our proposed method (Bayesian Composite Tobit Q Reg), the new hierarchical prior distribution and the Likelihood function of (ALD) for the error that is proposed by (Kozumi and Kobayashi (2011))[49] will produce full conditional posterior distributions. These complete conditional posterior distributions are informative for building a strong and efficient MCMC algorithm for our proposed method in order to achieving the coefficients estimation and variable selection in composite Tobit Q Reg model with a high accuracy.

- Our MCMC algorithm was presented for a Bayesian Composite Tobit Q Reg was very stable, this is clear from results of multivariate potential scale reduction factor (MPSRF) is computed via simulation1 at five type of error distributions, where it becomes stable and close to one after 3000 iterations. This shows that the convergence of the full conditional posterior distributions for the algorithm was very quick and the chain mixing was good.

In this thesis, we developed a simple and efficient MCMC algorithm based on computation technique for composite Tobit Q Reg based on a mixture of an exponential and a scaled normal distribution of the asymmetric Laplace distribution. Simulation studies show that our proposed method is effective in coefficient estimation ar different error distributions. Our proposed method (Bayesian Composite Tobit Q Reg) has a outperformance on other methods. We see performs of our proposed method with mixture error distributions that are better than non-mixture error distributions.

After assessing of our proposed method by using simulation approach, we will evaluate our proposed method (Bayesian composite Tobit Q Reg) with real dataset. We used the labour force participation data available within AER package in R, introduced by (Mroz (1987))[6] .The dataset consists of n = 753 observations of which 325 are censored observations.These data contain the one response variable (wife's hours of work in 1975 (hours)) and six covariates. Our proposed method has a good performance compared with other method with high censoring level in response variable. Therefore, our proposed method (Bayesian composite Tobit Q Reg) has a good performance with real dataset. Our new algorithm belonging to our method (Bayesian composite Tobit Q Reg) was efficient and it represented a simple way to achieve variable selection via computation of the relative importance to each covariates in our model composite Tobit Q Reg model.

6.2 . APPLIED CONCLUSIONS.

The proposed methods (new Bayesian Lasso Tobit quantile regression and Bayesian Composite Tobit Quantile Regression) are applied in in financial data for modelling of the relationship between response variable (Iraqi bank investments) and a set of independent variables. Via different Tobit quantile levels and groups of Tobit quantile levels respectively from our results, we reached to a set of general conclusions:

- From the results of Pseudo-R square, the thirtieth Tobit quantile regression line which belongs to ($\theta_{30} = 0.99$) was the best from all the Tobit quantile regression lines used to represent the studied data. Where the Pseudo-R square belongs to Tobit Q Reg model at $\theta_{30} = 0.99$ is equal 0.573283, this means that 57.32 % of the variation in censored response variable can be explained by a set of independent variables(x_1 :Banking Deposit ,x_2 Banking profits ,x_3 Bank capital ,x_4 :Bank reserves ,x_5 :Banking Loans , x_6 : advertising expenditures ,x_7 :Age of bank ,x_8 Number of Banks Branches ,x_9: Bad debt). Although, it is the most powerful line for interpreting of the data under study compared with the rest of the Tobit quantile regression lines, it does not have a high strength in interpreting that data. Therefore, we used Tobit quantile regression line which can interpret the data under study strongly, but the process of identifying is a very difficult one.
- The results are showed that the all composite Tobit quantile models which are belong to six groups of Tobit quantile levels have a high ability in explanation of the data studied. This clear from the Pseudo R-square results.

- The independent variables (x_1 :banking Deposit, x_2: Banking profits, x_4:Bank reserves) have a statistically significant effect on the response variable (Iraqi bank investments) via all groups of Tobit quantile levels. Also the independent variables (x_3: Bank capital, x_7:Age of bank, x_8:Number of Banks Branches, x_9: Bad debt) have a statistically significant effect on the response variable (Iraqi bank investments) via majority of groups of Tobit quantile levels . But, there are two independent variables (x_5:Banking Loans, x_6:Advertising Expenses) are insignificant in the response variable (Iraqi bank investments) via all groups of Tobit quantile levels.
- At six Groups of Tobit Quantile levels, there is a set of independent variables which are active in constructing composite Tobit Q Reg model via different Groups of Tobit Quantile levels as follows: At first group (H=5), there are six independent variables (x_1: Banking Deposits, x_2: Banking profits, x_3: Bank capital, x_6 : Advertising Expenses, x_8 :Number of Bank Branches, x_9: Bad debt) important in this model. In second group (H=10), there are six independent variables (x_1: Banking Deposits, x_2:Banking profits, x_3: Bank capital, x_6:Advertising Expenses, x_8 :Number of Bank Branches, x_9: Bad debt) effective in building this model. In third group (H=15), there are seven independent variables (x_1: Banking Deposits, x_2: Banking profits, x_3: Bank capital, x_4: Bank reserves,x_6 : Advertising Expenses,x_8 :Number of Bank Branches,x_9: Bad debt, x_9: Bad debt) strong in structure this model. In fourth group (H=20), there are eight independent variables (x_1: Banking Deposits, x_2: Banking profits, x_3: Bank capital, x_4: Bank reserves, x_6 : Advertising Expenses, x_7: Age of the Bank, x_8 :Number of Bank Branches, x_9: Bad debt) strong in structure this model. In fifth group (H=25), there are nine independent variables (x_1 :Banking Deposit ,x_2 Banking profits ,x_3 Bank capital ,x_4 :Bank reserves ,x_5 :Banking Loans , x_6 :

advertising expenditures ,x_7 :Age of bank ,x_8 Number of Banks Branches ,x_9: Bad debt) active in constructing our model. At sixth group (H=30), there are nine independent variables (x_1 :Banking Deposit ,x_2 Banking profits ,x_3 Bank capital ,x_4 :Bank reserves ,x_5 :Banking Loans , x_6 : advertising expenditures ,x_7 :Age of bank ,x_8 Number of Banks Branches ,x_9: Bad debt) which have a high relative importance in building our model.

- The independent variables (x_1: Banking Deposits, x_2: Banking profits, x_3: Bank capital, x_6 : Advertising Expenses, x_8 :Number of Bank Branches, x_9: Bad debt) are very strong in modelling the relationship with response variable (Iraqi banks investments) via all groups of Tobit Quantile levels. The rest independent variables (x_4 :Bank reserves, x_5:Banking Loans, x_7:Age of bank) are strong in modelling the relationship with response variable (Iraqi banks investments) via majority of groups of Tobit Quantile levels. The following table shows the speech above by briefly:

Table 6.1: Summary of status of independent variables via six groups of Tobit Quantile levels.

Independent variables / the groups	x_1: Banking Deposits	x_2: Banking profits	x_3 Bank capital	x_4 :Bank reserves	x_5 :Banking Loans	x_6 : advertising expenditures	x_7 :Age of bank	x_8 Number of Banks Branches	x_9: Bad debt
H=5	Active	Active	Active	Inactive	Inactive	Active	Inactive	Active	Active
H=10	Active	Active	Active	Inactive	Inactive	Active	Inactive	Active	Active
H=15	Active	Active	Active	Active	Inactive	Active	Inactive	Active	Active
H=20	Active	Active	Active	Inactive	Active	Active	Active	Active	Active
H=25	Active	Active	Active	Active	Active	Active	Active	Active	Active
H=30	Active	Active	Active	Active	Active	Active	Active	Active	Active

- For determining of the strength and weakness of the independent variables in the Tobit Q Reg model via different Tobit quantile levels by our proposed method (New Bayesian Tobit Q Reg) as follows:

x_1: Banking Deposits : The Larger probability value to this variable in Tobit Q Reg model at $\theta_{28} = 0.92$. Where ,its probability value is (0.839) greater than 0.5 . So, it is very important in building Tobit Q Reg model at $\theta_{28} = 0.92$. The smaller probability value to this variable in Tobit Q Reg model at $\theta_{23} = 0.79$. Where ,its probability value is (0.599) greater than 0.5 . Also, it is very important in building Tobit Q Reg model at $\theta_{28} = 0.92$. in generally, x_1: Banking Deposits is active in Tobit Q Reg model at all Tobit quantile levels. We cannot cancel this variable from our model.

x_2: Banking profits : The Larger probability value to this variable in Tobit Q Reg model at $\theta_{28} = 0.92$. Where ,its probability value is (0.853) greater than 0.5 . So, it is very active in building Tobit Q Reg model at $\theta_{28} = 0.92$. The smaller probability value to this variable in Tobit Q Reg model at $\theta_1 = 0.01$. Where ,its probability value is (0.413) less than 0.5. So, it is ineffective in building Tobit Q Reg model at $\theta_1 = 0.01$. In generally, x_2: Banking profits is active in Tobit Q Reg model at majority Tobit quantile levels. We can depend on this variable in modelling our model.

x_3 Bank capital: The Larger probability value to this variable in Tobit Q Reg model at $\theta_{30} = 0.99$. Where ,its probability value is (0.797) greater than 0.5 . So, it is strong in building Tobit Q Reg model at $\theta_{30} = 0.99$. The smaller probability value to this variable in Tobit Q Reg model at $\theta_{19} = 0.66$. Where ,its probability value is (0.298) less than 0.5. So, it is a weak in building Tobit Q Reg model at $\theta_{19} = 0.66$. In generally, x_3 Bank capital is unimportant in Tobit Q Reg model at majority Tobit quantile levels. We can ignore this variable from structure our model.

$\boldsymbol{x_4}$: **Bank reserves:** The Larger probability value to this variable in Tobit Q Reg model at $\theta_3 = 0.08$. Where ,its probability value is (0.778) greater than 0.5 . So, it is active in constructing a Tobit Q Reg model at $\theta_3 = 0.08$. The smaller probability value to this variable in Tobit Q Reg model at $\theta_{21} = 0.74$. Where ,its probability value is (0.309) less than 0.5. So, it is inactive in building a Tobit Q Reg model at $\theta_{21} = 0.74$. In generally**,** x_4: Bank reserves is a strong in a Tobit Q Reg model at majority Tobit quantile levels. We cannot delete this variable from structure our model.

$\boldsymbol{x_5}$ **:Banking Loans**: The Larger probability value to this variable in Tobit Q Reg model at $\theta_{30} = 0.99$. Where ,its probability value is (0.802) greater than 0.5 . So, it is active in building a Tobit Q Reg model at $\theta_{30} = 0.99$. The smaller probability value to this variable in Tobit Q Reg model at $\theta_{12} = 0.42$. Where ,its probability value is (0.332) less than 0.5. So, it is a weak in building a Tobit Q Reg model at $\theta_{12} = 0.42$. In generally**,** x_5 :Banking Loans is very active in a Tobit Q Reg model at majority Tobit quantile levels. We cannot ignore it from building our model.

$\boldsymbol{x_6}$ **: Advertising Expenses**: The Larger probability value to this variable in Tobit Q Reg model at $\theta_{30} = 0.99$. Where ,its probability value is (0.811) greater than 0.5 . So, it is strong in building a Tobit Q Reg model at $\theta_{30} = 0.99$. The smaller probability value to this variable in Tobit Q Reg model at $\theta_{19} = 0.66$. Where ,its probability value is (0.261) less than 0.5. So, it is a trifle in constructing a Tobit Q Reg model at $\theta_{19} = 0.66$. In generally**,** x_6 : Advertising Expenses is very a weak in a Tobit Q Reg model at majority Tobit quantile levels. We can delete it from building our model.

$\boldsymbol{x_7}$ **:Age of bank**: The Larger probability value to this variable in Tobit Q Reg model at $\theta_{30} = 0.99$. Where ,its probability value is (0.822) greater than 0.5 . So, it is active in building a Tobit Q Reg model at $\theta_{30} = 0.99$. The smaller probability value to this variable in Tobit Q Reg model at $\theta_{18} = 0.63$. Where ,its probability value is (0.339) less than 0.5. So, it is ineffective in building a Tobit Q Reg model at $\theta_{18} = 0.63$. In generally**,** x_7 :Age of bank is a strong in Tobit Q Reg model at majority Tobit quantile levels. We cannot ignore it from structure our model.

$\boldsymbol{x_8}$ **Number of Banks Branches** :The Larger probability value to this variable in Tobit Q Reg model at $\theta_{30} = 0.99$. Where ,its probability value is (0.828) greater than 0.5 . So, it is effective in structure a Tobit Q Reg model at $\theta_{30} = 0.99$. The smaller probability value to this variable in Tobit Q Reg model at $\theta_{17} = 0.60$. Where ,its probability value is (0.325) less than 0.5. So, it is a trifle in building a Tobit Q Reg model at $\theta_{17} = 0.60$. In generally**,** x_8 Number of Banks Branches is very a weak in a Tobit Q Reg model at majority Tobit quantile levels. We can cancel it from structure our model.

$\boldsymbol{x_9}$**: Bad debt:** The Larger probability value to this variable in Tobit Q Reg model at $\theta_{30} = 0.99$. Where ,its probability value is (0.875) greater than 0.5 . So, it is very strong in building a Tobit Q Reg model at $\theta_{30} = 0.99$. The smaller probability value to this variable in Tobit Q Reg model at $\theta_{12} = 0.42$. Where ,its probability value is (0.591) greater than 0.5 . also, it is a strong in building a Tobit Q Reg model at $\theta_{12} = 0.42$. In generally**,** x_9: Bad debt is very strong in Tobit Q Reg model at all Tobit quantile levels. We cannot delete this variable from structure our model.

- The independent variables (x_1: Banking Deposits,x_9: Bad debt) have probability value greater than 0.5 in Tobit Q Reg model at all Tobit quantile levels. Therefore these independent variables is very active in our model. And the independent variables (x_2: Banking profits,x_5 :Banking Loans, x_7 :Age of bank) have probability value greater than 0.5 in Tobit Q Reg model at majority Tobit quantile levels. Therefore these independent variables is very strong in our model. But the independent variables (x_3: Bank capital, x_6 : Advertising Expenses, x_8 :Number of Bank Branches) have probability value less than 0.5 in Tobit Q Reg model at majority Tobit quantile levels. Therefore these independent variables is very a weak in our model, we can cancel them from our model.

- For determining of the strength and weakness of the independent variables in the composite Tobit Q Reg model via six groups of composite Tobit quantile levels by our proposed method (Bayesian composite Tobit Q Reg) as follows:

$\boldsymbol{x_1}$**: Banking Deposits** : The Larger probability value to this variable in composite Tobit Q Reg model at ten composite Tobit quantile levels [H=10] . Where ,its probability value is (0.917) greater than 0.5 . So, it is very active in building a composite Tobit Q Reg model at [H=10] . The smaller probability value to this variable in composite Tobit Q Reg model at five composite Tobit quantile levels [H=5] . Where ,its probability value is (0.893) greater than 0.5 . Also, it is very active in building Tobit Q Reg model at [5] . In generally, x_1: Banking Deposits is a strong in composite Tobit Q Reg model at all six groups of composite Tobit quantile levels . We cannot cancel this variable from our model.

$\boldsymbol{x_2}$**: Banking profits :**The Larger probability value to this variable in composite Tobit Q Reg model at twenty five composite Tobit quantile levels [H=25] . Where ,its probability value is (0.907) greater than 0.5 . So, it is very strong in structure a composite Tobit Q Reg model at [25] . The smaller probability value to this variable in composite Tobit Q Reg model at five composite Tobit quantile levels [H=5] . Where ,its probability value is (0.880) greater than 0.5 . Also , it is effective in building a Tobit Q Reg model at [5] . In generally, x_2: Banking profits has a high relative importance in composite Tobit Q Reg model at all six groups of composite Tobit quantile levels. We can depend on this variable in modelling our model strongly .

$\boldsymbol{x_3}$ **Bank capital**: The Larger probability value to bank capital in composite Tobit Q Reg model at thirty composite Tobit quantile levels [H=30] . Where ,its probability value is (0.703) greater than 0.5 . So, it is effective in constructing a composite Tobit composite Q Reg model at$[\mathrm{H} = 30]$. The smaller probability value to bank capital in composite Tobit Q Reg model at five composite Tobit quantile levels [H=5] . Where ,its probability value is (0.642) less than 0.5. Also, the bank capital is very strong in building a composite Tobit Q Reg model at[5]. In generally, x_3 Bank capital is important in composite Tobit Q Reg model at all six groups of composite Tobit quantile levels. We cannot ignore this variable from structure our model.

$\boldsymbol{x_4}$: **Bank reserves:** The Larger probability value to this variable in composite Tobit Q Reg model at twenty composite Tobit quantile levels [H=20]. Where ,its probability value is (0.617) greater than 0.5 . So, it is effective in building a composite Tobit Q Reg model at $[H = 20]$. The smaller probability value to this variable in composite Tobit Q Reg model at five composite Tobit quantile levels [H=5] . Where ,its probability value is (0.346) less than 0.5. So, it is active in building a composite Tobit Q Reg model at $[\mathrm{H} = 5]$. In generally, x_4: Bank reserves is a strong in a Tobit Q Reg model at majority six groups of composite Tobit quantile levels . We cannot delete this variable from structure our model.

$\boldsymbol{x_5}$ **:Banking Loans**: The Larger probability value to banking loans in composite Tobit Q Reg model at twenty five composite Tobit quantile levels [H=25]. Where ,its probability value is (0.667) greater than 0.5 . So, banking loans is active in building a Tobit Q Reg model [25]. The smaller probability value to this variable in composite Tobit Q Reg model at twenty composite Tobit quantile levels [H=20]. Where ,its probability value is (0.306) less than 0.5. So, it is a very weak in building a composite Tobit Q Reg model at [H=20] . In generally, x_5 :Banking Loans is very a weak in a composite Tobit Q Reg model at majority six groups of composite Tobit quantile levels . We can ignore this variable from structure our model.

x_6 : Advertising Expenses: The Larger probability value to advertising expenses in composite Tobit Q Reg model at thirty composite Tobit quantile levels [H=30]. Where ,its probability value is (0.853), greater than 0.5 . So, it is strong in building a composite Tobit Q Reg model at [30]. The smaller probability value to this variable in composite Tobit Q Reg model at five composite Tobit quantile levels [H=5]. Where ,its probability value is (0.802) less than 0.5. Also , it is a very strong in constructing a composite Tobit Q Reg model at [H=5]. In generally**,** x_6 : Advertising Expenses is a very strong in a composite Tobit Q Reg model at majority Tobit quantile levels. We can delete it from building our model, at all six groups of composite Tobit quantile levels . We cannot omit this variable from structure our model.
x_7 :Age of bank: The Larger probability value to age of bank in composite Tobit Q Reg model at twenty composite Tobit quantile levels [H=20]. Where ,its probability value is (0.684) greater than 0.5 . So, it is active in building a composite Tobit Q Reg model at [H=20]. The smaller probability value age of bank in composite Tobit Q Reg model at ten composite Tobit quantile levels [H=10]. Where ,its probability value is (0.324) less than 0.5. So, it is inactive in building a composite Tobit Q Reg model at [10]. In generally**,** x_7 :Age of bank is a active in composite Tobit Q Reg model at majority six groups of composite Tobit quantile levels.
x_8 Number of Banks Branches :The Larger probability value to this variable in composite Tobit Q Reg model at twenty five composite Tobit quantile levels [H=25]. Where ,its probability value is (0.903) greater than 0.5 . So, it is very effective in structure a composite Tobit Q Reg model at [25]. The smaller probability value to this variable in composite Tobit Q Reg model at twenty composite Tobit quantile levels [H=20]. Where ,its probability value is (0.865) less than 0.5. Also , it is a strong in building a composite Tobit Q Reg model at[H=20] . In generally**,** x_8 Number of Banks Branches is very strong in a composite Tobit Q Reg model, at all six groups of composite Tobit quantile levels . We cannot omit this variable from structure our model.
x_9: Bad debt: The Larger probability value to bad debt in composite Tobit Q Reg model at twenty five composite Tobit quantile levels [H=25].Where ,its probability value is (0.910) greater than 0.5 . So, it is very strong in constructing a composite Tobit Q Reg model at [25] . The smaller probability value to bad debt in composite Tobit Q Reg model at five composite Tobit quantile levels [H=5]. Where ,its probability value is (0.892) greater than 0.5 . Also, it is a strong in building a Tobit Q Reg model at [5]. In generally**,** x_9: Bad debt is very strong in composite Tobit Q Reg model at all six groups of composite Tobit quantile levels . We cannot ignore this variable from structure our model.

- The independent variables (x_1: Banking Deposits, x_2: Banking profits, x_3: Bank capital, x_6 : Advertising Expenses, x_8 :Number of Bank Branches,x_9: Bad debt) have probability value greater than 0.5 in composite Tobit Q Reg model at all six groups of composite Tobit quantile levels. So, these independent variables is very strong in our model. And the independent variables (x_4: Bank reserves, x_7 :Age of bank) have probability value greater than 0.5 in composite Tobit Q Reg model at majority six groups of composite Tobit quantile levels. Therefore these independent variables is very strong in our model. But the independent variable (x_5 :Banking Loans) have probability value less than 0.5 in composite Tobit Q Reg model at majority Tobit quantile levels. Therefore these independent variables is very a weak in our model, we can omit it from our model.

6.3. FUTURE RESEARCHES

Our proposed methods have a good possibility in variable selection and coefficient estimation in a set of regression models. Therefore, these methods can be easily extended to several ways. As follows:

- Our proposed method new Bayesian Lasso quantile regression can be easily extended to new Bayesian adaptive Lasso quantile regression via using a scale mixture uniform instead of scale mixture normal where a scale mixture of uniform will create a new hierarchical Bayesian formulation of adaptive Lasso. The expected new Bayesian treatment leads to a simple and efficient Gibbs sampler conditional posterior distributions. Simulation approaches and real data are used to test the performance of proposed method compared with other methods in the same field. The expected proposed method considers a new addition in variable selection and prediction accuracy in quantile regression model at different quantile levels.

- Our method new Bayesian Lasso Tobit Q Reg can be extended to new Bayesian adaptive Lasso Tobit quantile regression with a new hierarchical prior distribution via using a scale mixture uniform which generates a new hierarchical Bayesian for full conditional posterior distribution. The expected a new hierarchical Bayesian give us attractive and informative Gibbs sampler. Also the expected, new Bayesian adaptive Lasso Tobit Q Reg is very effective in coefficient estimation and variable selection in Tobit quantile regression model. Simulation scenarios and real dataset will used to evaluating new Bayesian adaptive Lasso Tobit Q Reg compared with other methods in the same field.

- Our proposed method new Bayesian Lasso Tobit Q Reg model can be easily extended this method to Bayesian Lasso composite Tobit Q Reg with a mixture uniform prior scale through assigning an independent scale-mixture of uniform prior distributions to parameters of model. This suggestion creates a new structure of prior distributions to all parameters of model. This new stricture generates informative conditional posterior distributions which are lead to attractive and efficient Gibbs sampling algorithm, and this algorithm is very robust until with the response variable has much censored data. To assessing this proposed method the simulation approach and rea data will used.

- Binary quantile regression was developed by Manski (1975, 1985) and employed in classification, indicating the drawbacks in frequent processes given the difficulty of optimization to estimate the parameters and the problem of computing confidence interval to the parameters. Kordas (2006) studied models with binary response variable by quantile regression and concluded that this approach drives to good classification. Bayesian approach was adopted by (Benoit et al.(2012))[11] in order to avoid the drawback that mention above by setting some assumptions on the error term. Miguéis et al. (2013) considered the approach proposed by (Benoit et al. (2012))[11] to evaluate the credit risk and it was modelled by binary quantile regression. The novel in this proposed study is the Bayesian hierarchical model to estimate the coefficients of the composite quantile regression model when the response variable is binary. In order to select variables, in binary composite quantile regression Lasso and the adoptive Lasso penalty is derived in a Bayesian framework.

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