Universitatea din Craiova

Şcoala Doctorală de Ştiinţe

Domeniul Matematică

Rezumat teză de doctorat:

A Variational Analysis of Some Classes of Integral and Differential Equations: Eigenvalue Problems and Torsional Creep Problems

Maria Fărcăşeanu

Conducător de doctorat:
Prof. Univ. Dr. *Mihai Mihăilescu*

Craiova
2018

1 Introduction

A partial differential equation (PDE) is an equation involving an unknown function of two or more variables and certain of its partial derivatives. PDE's appear frequently in all domains, such as physics, mechanics and engineering. In fact, whenever we have an interaction between some independent variables, we attempt to find functions using these variables and to shape a multitude of processes by developing equations for these functions. Consequently, due to the rich variety of phenomena which can be modeled by PDE's, there is no general theory known concerning the solvability of all of them.

There are many methods to solve PDE's, each method being applicable to a certain class of equations. Solving a given PDE depends in large part on the particular structure of the problem at hand. It is considered that a given problem is well-posed if it has a solution which is unique and stable (i.e. the solution depends continuously on the data given in the problem). There are many different definitions of the solution for a PDE. The most natural notion of solution arises when all the derivatives which appear in the statement of the PDE exist and are continuous, although maybe certain higher derivatives do not exist. This kind of solutions are called "*classical*" solutions. On the other hand, there are functions for which the derivatives may not all exist, but which satisfy the equation in some precisely defined sense. These functions are known in the literature as "*weak*" solutions and they are most often used in the analysis of PDE's. However, even in situations where an equation has differentiable solutions, it is often convenient to prove first the existence of weak solutions and only later to show that those solutions are in fact smooth enough.

In general, the (weak) solutions can be found as critical points of the corresponding variational functionals defined on an appropriate function space dictated by the data of the problem. The simplest way to obtain such a critical point, is to look for a global extremum, which in most of the cases is a global minimum. If the functional has good properties, such as the smoothness or the boundedness, the existence of the minimum points can be obtained by applying direct methods in the calculus of variations. Otherwise, for example, the lack of smoothness can be tackled by a reformulation of the problem as a variational inequality, or if the functional is unbounded, there exist some minimization techniques that can still be profitably used, by constraining the functional on a set where it is bounded from below. Typical examples of such techniques are minimization on Spheres, or on the Nehari manifold.

Some of the fundamental problems in mathematical physics are, probably, the eigenvalue problems for elliptic PDE's. The analysis of such equations involves, in general, energy methods which are based on the critical point theory that has been mentioned previously. For example, the eigenvalue problem for the $p-$Laplace operator subject to zero Dirichlet boundary condition, i.e.

$$\begin{cases} -\Delta_p u = \lambda |u|^{p-2} u & \text{in} \quad \Omega \\ u = 0 & \text{on} \quad \partial\Omega\,, \end{cases} \tag{1}$$

where Ω is a bounded domain in $\mathbb{R}^N$, $p \in (1, \infty)$ and $\Delta_p u := \operatorname{div}(|\nabla u|^{p-2}\nabla u)$ stands for the $p-$Laplacian, has been studied extensively along the time and many interesting results have been obtained. If $p = 2$, problem (1) becomes the eigenvalue problem for the Laplacian, that is

$$\begin{cases} -\Delta u = \lambda u & \text{in} \quad \Omega \\ u = 0 & \text{on} \quad \partial\Omega \,, \end{cases}$$

and it is well-known that all the eigenvalues are positive and form an increasing and unbounded sequence $0 < \lambda_1 < \lambda_2 \leq \lambda_3 \leq ...$ such that $\lambda_n \to +\infty$ as $n \to \infty$. Moreover, in this particular case, all the eigenvalues have finite multiplicities and the first one is simple. For $p \neq 2$ and $N \geq 2$, the complete description of the set of all eigenvalues is an open problem. It is known that the Ljusternik-Schnirelman theory ensures the existence of an infinite sequence of positive eigenvalues of problem (1), but in general this theory does not provide all eigenvalues. However, it can be shown, the existence of a principal eigenvalue, $\lambda_1(p)$, that is the smallest of all possible eigenvalues λ, which can be characterized from a variational point of view in the following manner

$$\lambda_1(p) := \inf_{u \in C_0^\infty(\Omega) \setminus \{0\}} \frac{\int_\Omega |\nabla u|^p \, dx}{\int_\Omega |u|^p \, dx}.$$

Moreover, $\lambda_1(p)$ is simple, isolated and the corresponding eigenfunctions are minimizers of $\lambda_1(p)$, that do not change sign in Ω. Also, it was showed that, if $u_p > 0$ is an eigenfunction associated to $\lambda_1(p)$, then there exists a subsequence of $\{u_p\}$, which converges uniformly in Ω, when $p \to \infty$, to a nontrivial and nonnegative solution, defined in the viscosity sense, of the limiting problem

$$\begin{cases} \min\{|\nabla u| - \Lambda_\infty u, -\Delta_\infty u\} = 0 & \text{in} \quad \Omega \\ u = 0 & \text{on} \quad \partial\Omega \,, \end{cases}$$

where Δ_∞ is the $\infty-$Laplace operator, which on sufficiently smooth functions $u : \Omega \to \mathbb{R}$ is given by $\Delta_\infty u := \sum_{i,j=1}^N \frac{\partial u}{\partial x_i} \frac{\partial u}{\partial x_j} \frac{\partial^2 u}{\partial x_i \partial x_j} = \langle D^2 u \nabla u, \nabla u \rangle$ and

$$\Lambda_\infty := \frac{1}{\max\limits_{x \in \Omega} dist(x, \partial\Omega)}.$$

Motivated by the above results that have been obtained in the case of the $p-$Laplace operator, the first part of the thesis (Chapter 2) is devoted to the study of various eigenvalue problems which involve different types of elliptic partial differential operators or integral operators. For instance, we consider an anisotropic version of the $p-$Laplacian, that is the $(p, q)-$Laplace operator, defined by

$$\Delta_{p,q} u := \mathrm{div}_x \left(|\nabla_x u|^{p-2} \nabla_x u \right) + \mathrm{div}_y \left(|\nabla_y u|^{q-2} \nabla_y u \right) \,,$$

where we have denoted by $\nabla_x u$ and $\nabla_y u$ the derivatives of u with respect to the first L variables and with respect to the last M variables $(L + M = N)$ and a fractional version of the $p-$Laplacian, called fractional $(s, p)-$Laplacian, given by

$$(-\Delta_p)^s u(x) := 2 \lim_{\epsilon \searrow 0} \int_{|x-y| \geq \epsilon} \frac{|u(x) - u(y)|^{p-2}(u(x) - u(y))}{|x - y|^{N+sp}} \, dy, \ x \in \mathbb{R}^N \,,$$

where $1 < p < \infty$ and $0 < s < 1$. To each of these operators, we associate adequate eigenvalue problems and we characterize their spectrum using methods based on critical point theory. Besides that, in Chapter 2, we study the continuity of the first eigenvalue with respect to a parameter, for a family of degenerate eigenvalue problems and, in the end, we give a maximum principle for a class of first order differential operators, using as starting point an eigenvalue problem for elliptic operators involving variable exponent growth conditions.

The second part of the thesis (Chapter 3) is devoted to the study of some PDE's that are connected with the concept of "torsional creep". This phenomenon is explained as being the permanent plastic deformation of a material subject to a torsional moment for an extended period of time and at sufficiently high temperature. The modelling of such a phenomenon is related to inhomogeneous problems of the type

$$\begin{cases} -\Delta_p u = 1 & \text{in} \quad \Omega \\ u = 0 & \text{on} \quad \partial\Omega\,, \end{cases} \tag{2}$$

when $p \to \infty$. It is known that problem (2) possesses a unique solution, u_p, which uniformly converges to function $\text{dist}(\cdot, \partial\Omega)$ (that is the distance function to the boundary of Ω), as $p \to \infty$. Note that, the limit case is of special interest in applications, since it models the perfect plastic torsion. In this chapter, our aim will be the study of the asymptotic behaviour of some families of solutions for different equations, which represent extensions of the classical torsional creep problem (2).

2 Main results

The thesis is structured into 3 chapters (Chapters 2-4). Chapters 2 and 3 represent the main body of the thesis, presenting the main results of our research. Chapter 4 contains some open problems on the topic of the thesis that represent the starting point for our further research. In the following we describe in brief the main results from the thesis.

Throughout the thesis, we consider that Ω is a bounded domain from $\mathbb{R}^N$ with smooth boundary $\partial\Omega$.

Chapter 2 is devoted to the study of various eigenvalue problems, which involve different types of differential or integral operators. In this chapter, λ denotes a real parameter, which will be called an eigenvalue of a problem if that problem has a nontrivial solution defined in a variational way. This chapter contains 4 sections (Sections 2.1-2.4).

Section 2.1 (based on paper [1]) is concerned with the study of an eigenvalue problem involving an anisotropic (p, q)-Laplacian. More precisely, if L and M are two positive integers, such that $L+M = N$, then for each two real numbers p and q, satisfying $1 < p < q < \infty$, and each smooth function $u : \Omega \to \mathbb{R}$, we define the *anisotropic* (p, q)*-Laplacian* by

$$\Delta_{p,q} u := \text{div}_x \left(|\nabla_x u|^{p-2} \nabla_x u \right) + \text{div}_y \left(|\nabla_y u|^{q-2} \nabla_y u \right) \,,$$

where we have denoted by $\nabla_x u$ and $\nabla_y u$ the derivatives of u with respect to the first L variables and with respect to the last M variables, respectively, that is,

$$\nabla_x u = \left(\frac{\partial u}{\partial x_1}, \frac{\partial u}{\partial x_2}, ..., \frac{\partial u}{\partial x_L} \right) \quad \text{and} \quad \nabla_y u = \left(\frac{\partial u}{\partial y_1}, \frac{\partial u}{\partial y_2}, ..., \frac{\partial u}{\partial y_M} \right) .$$

The goal of Section 2.1, is to study the existence of nontrivial solutions of the following anisotropic eigenvalue problem

$$\begin{cases} -\Delta_{p,q} u = \lambda |u|^{q-2}u, & \text{in } \Omega \\ u = 0, & \text{on } \partial\Omega\,. \end{cases} \tag{3}$$

The main result of this section is given by the following theorem (see Theorem 2.1 in the thesis).

Theorem 1. *Assume that* $1 < p < q < \infty$ *and either* $p \geq N$ *or*

$$\frac{L}{p} + \frac{M}{q} > 1 \quad \text{and} \quad \frac{L}{p} - \frac{L}{q} < 1\,.$$

Then the set of eigenvalues of problem (3) is given exactly by the open interval $(\mu_1(q), \infty)$*, where*

$$\mu_1(q) := \inf_{u \in W_0^{1,p,q}(\Omega)\setminus\{0\}} \frac{\displaystyle\int_\Omega |\nabla_y u|^q}{\displaystyle\int_\Omega |u|^q}\,.$$

In Section 2.2 (based on papers [2] and [6]), we study two eigenvalue problems involving an integral operator. This section is divided into two subsections: 2.2.1 and 2.2.2. In order to present the main results from these subsections, we define for each $p \in (1, \infty)$ and $s \in (0, 1)$, the *fractional* (s, p)*-Laplace operator* by

$$(-\Delta_p)^s u(x) := 2 \lim_{\epsilon \searrow 0} \int_{|x-y|\geq\epsilon} \frac{|u(x) - u(y)|^{p-2}(u(x) - u(y))}{|x - y|^{N+sp}}\, dy, \;\; x \in \mathbb{R}^N\,.$$

The common eigenvalue problem associated to the fractional (s, p)-Laplace operator is given by

$$\begin{cases} (-\Delta_p)^s u(x) = \lambda |u(x)|^{p-2}u(x), & x \in \Omega \\ u(x) = 0, & \text{for } x \in \mathbb{R}^N\setminus\Omega\,. \end{cases} \tag{4}$$

It is well known that the first eigenvalue of (4), denoted by $\lambda_1(s, p)$, can be characterized from a variational point of view by

$$\lambda_1(s, p) := \inf_{u \in C_0^\infty(\Omega)\setminus\{0\}} \frac{\displaystyle\int_{\mathbb{R}^N}\int_{\mathbb{R}^N} \frac{|u(x) - u(y)|^p}{|x - y|^{N+sp}}\, dx\, dy}{\displaystyle\int_{\mathbb{R}^N} |u|^p\, dx}\,. \tag{5}$$

In Subsection 2.2.1, we investigate the problem

$$\begin{cases} (-\Delta_p)^s u(x) = \lambda f(x, u(x)), & \text{for } x \in \Omega \\ u(x) = 0, & \text{for } x \in \mathbb{R}^N\setminus\Omega\,, \end{cases} \tag{6}$$

where function $f : \Omega \times \mathbb{R} \to \mathbb{R}$ is given by

$$f(x, t) = \begin{cases} h(x, t), & \text{if } t \geq 0, \\ |t|^{p-2}t, & \text{if } t < 0\,. \end{cases} \tag{7}$$

Function $h : \Omega \times [0, \infty) \to \mathbb{R}$ is a Caratheodory function, satisfying the following hypotheses

(H1) there exists a positive constant $C \in (0,1)$ such that $|h(x,t)| \leq Ct^{p-1}$, for any $t \geq 0$ and a.e. $x \in \Omega$;

(H2) there exists $t_0 > 0$, such that $H(x,t_0) = \int_0^{t_0} h(x,s)\, ds > 0$ for a.e. $x \in \Omega$;

(H3) $\lim\limits_{t\to\infty} \frac{h(x,t)}{t^{p-1}} = 0$, uniformly in Ω.

The main result of this subsection is the following (see Theorem 2.2 in the thesis).

Theorem 2. *Assume that f is given by relation (7) and conditions (H1), (H2) and (H3) are fulfilled. Then, $\lambda_1(s,p)$ defined in (5), is an isolated eigenvalue of problem (6). Moreover, any $\lambda \in (0, \lambda_1(s,p))$ is not an eigenvalue of problem (6), but there exists $\mu_1 > \lambda_1(s,p)$, such that any $\lambda \in (\mu_1, \infty)$ is an eigenvalue of problem (6).*

Next, in Subsection 2.2.2, we study the following perturbed eigenvalue problem

$$\begin{cases} (-\Delta_p)^s u(x) + (-\Delta_q)^t u(x) = \lambda |u(x)|^{r-2} u(x), & \text{for } x \in \Omega \\ u(x) = 0, & \text{for } x \in \mathbb{R}^N \backslash \Omega\,, \end{cases} \tag{8}$$

where s, t, p and q are real numbers satisfying the assumption

$$0 < t < s < 1, \quad 1 < p < q < \infty, \quad s - \frac{N}{p} = t - \frac{N}{q}\,, \tag{9}$$

and $r \in \{p, q\}$. Our purpose is to determine all the parameters λ, for which problem (8) possesses nontrivial weak solutions. With that end in view, define

$$\lambda_1 := \begin{cases} \lambda_1(s,p) := \inf\limits_{u \in C_0^\infty(\Omega)\backslash\{0\}} \dfrac{\displaystyle\int_{\mathbb{R}^N}\int_{\mathbb{R}^N} \frac{|u(x)-u(y)|^p}{|x-y|^{N+sp}}\, dx\, dy}{\displaystyle\int_{\mathbb{R}^N} |u|^p\, dx}, & \text{if} \quad r = p \\[2ex] \lambda_1(t,q) := \inf\limits_{u \in C_0^\infty(\Omega)\backslash\{0\}} \dfrac{\displaystyle\int_{\mathbb{R}^N}\int_{\mathbb{R}^N} \frac{|u(x)-u(y)|^q}{|x-y|^{N+tq}}\, dx\, dy}{\displaystyle\int_{\mathbb{R}^N} |u|^q\, dx}, & \text{if} \quad r = q\,. \end{cases}$$

The main result of this subsection is given by the following theorem (see Theorem 2.3 in the thesis).

Theorem 3. *Assume condition (9) is fulfilled. Then the set of all real parameters λ for which problem (8) has at least a nontrivial weak solution is the interval (λ_1, ∞), with λ_1 defined above. Moreover, the weak solution could be chosen to be non-negative.*

In Section 2.3 (based on paper [3]), for each $\alpha \in [0,2)$, we consider the eigenvalue problem

$$\begin{cases} -\mathrm{div}(|x|^\alpha \nabla u) = \lambda u, & \text{for } x \in \Omega \\ u = 0, & \text{for } x \in \partial\Omega\,, \end{cases} \tag{10}$$

where $0 \in \Omega$ and the Rayleigh quotient corresponding to this equation

$$\frac{\displaystyle\int_\Omega |x|^\alpha |\nabla u|^2\, dx}{\displaystyle\int_\Omega u^2\, dx}\,.$$

The infimum of the above quotient among all smooth functions with zero boundary values, i.e.

$$\lambda_1(\alpha) := \inf_{u \in C_0^\infty(\Omega)\setminus\{0\}} \frac{\displaystyle\int_\Omega |x|^\alpha |\nabla u|^2 \, dx}{\displaystyle\int_\Omega u^2 \, dx},$$

is positive and gives the first eigenvalue of problem (10). Thus, we can define the function $\lambda_1 : [0, 2) \to (0, \infty)$. The main result of this section is given by the following theorem (see Theorem 2.4 in the thesis).

Theorem 4. *The function $\lambda_1 : [0, 2) \to (0, \infty)$ is continuous.*

The goal of Section 2.4 (based on paper [5]) is to present how a series of results obtained in connection with an eigenvalue problem involving variable exponents can be used in order to obtain a maximum principle, which complements the classical maximum principle for elliptic operators. The main result of this section is given by the following theorem (see Theorem 2.7 in the thesis).

Theorem 5. *Let $\overrightarrow{a} : \Omega \to \mathbb{R}^N$ be a vectorial function such that $\overrightarrow{a} \in C^1(\Omega; \mathbb{R}^N) \cap C(\overline{\Omega}; \mathbb{R}^N)$. Assume that there exists a positive constant $a_0 > 0$ such that*

$$\operatorname{div} \overrightarrow{a}(x) \geq a_0 > 0, \quad \forall\, x \in \overline{\Omega}. \tag{11}$$

If $p \in C^1(\Omega) \cap C(\overline{\Omega})$ is a solution of the differential inequality

$$\overrightarrow{a}(x) \cdot \nabla p(x) \leq 0, \quad \forall\, x \in \Omega, \tag{12}$$

then for each open set $U \subset \Omega$ the minimum of p in $\overline{U}$ is achieved on ∂U.

If $p \in C^1(\Omega) \cap C(\overline{\Omega})$ is a solution of the differential inequality

$$\overrightarrow{a}(x) \cdot \nabla p(x) \geq 0, \quad \forall\, x \in \Omega, \tag{13}$$

then for each open set $U \subset \Omega$ the maximum of p in $\overline{U}$ is achieved on ∂U.

If $p \in C^1(\Omega) \cap C(\overline{\Omega})$ is a solution of the PDE's

$$\overrightarrow{a}(x) \cdot \nabla p(x) = 0, \quad \forall\, x \in \Omega, \tag{14}$$

then for each open set $U \subset \Omega$ the maximum and minimum of p in $\overline{U}$ are achieved on ∂U.

Chapter 3 is divided into 3 sections and devoted to the study of some PDE's that are related with the concept of torsional creep.

In Section 3.1 (based on paper [7]), we continue to keep the connection with the previous chapter, by considering, for each integer $n \geq 1$, the family of eigenvalue problems

$$\begin{cases} -\Delta_{2n} u = \mu u, & \text{for } x \in \Omega \\ u = 0, & \text{for } x \in \partial\Omega \\ \|u\|_{L^2(\Omega)} = 1, \end{cases} \tag{15}$$

where $\Delta_{2n} u := \operatorname{div}(|\nabla u|^{2n-2} \nabla u)$ is the $2n$-Laplace operator and μ is a real number. The main result of this section is given by the following theorem (see Theorem 3.1 in the thesis).

Theorem 6. *For each integer $n \geq 1$ define*

$$\mu_1(n) := \inf_{u \in W_0^{1,2n}(\Omega)\setminus\{0\}} \frac{\displaystyle\int_\Omega |\nabla u|^{2n}\, dx}{\left(\displaystyle\int_\Omega u^2\, dx\right)^n}\,.$$

Then $\mu_1(n)$ is a positive real number, which gives the lowest eigenvalue of problem (15). Letting u_n be a corresponding positive eigenfunction, the sequence $\{u_n\}$ converges uniformly in Ω to $\|\delta\|^{-1}_{L^2(\Omega)}\delta$, where $\delta(x) := \inf_{y\in\partial\Omega}|x-y|$, $\forall\, x \in \Omega$, denotes the distance function to the boundary of Ω .

The goal of Section 3.2 (based on paper [4]), is to investigate the asymptotic behaviour of the family of solutions for the following family of equations

$$\begin{cases} -\mathrm{div}\left(\frac{\varphi_n(|\nabla u|)}{|\nabla u|}\nabla u\right) = \varphi_n(1) & \text{in} \quad \Omega \\ u = 0 & \text{on} \quad \partial\Omega\,, \end{cases} \tag{16}$$

where, for each integer $n > 1$, the mappings $\varphi_n : \mathbb{R} \to \mathbb{R}$ are odd, increasing homeomorphisms of class C^1 defined by

$$\varphi_n(t) := p_n|t|^{p_n-2}te^{|t|^{p_n}}, \quad \forall\, t \in \mathbb{R}\,, \tag{17}$$

where $p_n \in (1,\infty)$ are given real numbers, such that $\lim_{n\to\infty} p_n = +\infty$. The main result of this section is the following theorem (see Theorem 3.3 in the thesis).

Theorem 7. *Problem (16), with φ_n given by relation (17), has a unique variational solution for each integer $n > 1$, provided that $p_n \in [2,\infty)$, which is nonnegative in Ω, say u_n. Moreover, under the supplementary assumption that $\lim_{n\to\infty} p_n = \infty$, the sequence $\{u_n\}$ converges uniformly in Ω to the distance function to the boundary of Ω.*

In Section 3.3 (based on paper [8]), we consider $H : \mathbb{R}^N \to [0,\infty)$ a Finsler norm and $\alpha : \overline{\Omega} \times \mathbb{R} \to (0,\infty)$ a continuous function for which there exist two positive constants λ, Λ, such that

$$0 < \lambda \leq \alpha(x,t) \leq \Lambda < +\infty, \ \ \forall\, x \in \overline{\Omega}, \ \forall\, t \in \mathbb{R}. \tag{18}$$

For each real number $p \in (N,\infty)$, we consider the following problem

$$\begin{cases} -\mathrm{div}(\alpha(x,u)H(\nabla u)^{p-2}\mathcal{H}(\nabla u)) = f, & x \in \Omega, \\ u = 0, & x \in \partial\Omega, \end{cases} \tag{19}$$

where $f : \overline{\Omega} \to (0,\infty)$ is a given continuous function and $\mathcal{H} : \mathbb{R}^N \to \mathbb{R}^N$ is defined by

$$\mathcal{H}_i(\xi) := \frac{\partial}{\partial \xi_i}\left(\frac{1}{2}H(\xi)^2\right), \quad \forall\, \xi \in \mathbb{R}^N, \ \forall\, i \in \{1,...,N\}\,.$$

The main results of this section are given by the following theorems (see Theorem 3.5 and Theorem 3.6 in the thesis).

Theorem 8. *Assume that condition (18) is fulfilled. Then for each $p \in (N,\infty)$ problem (19) has a weak solution $u_p \in W_0^{1,p}(\Omega)$ such that $u_p(x) \geq 0$ for a.e. $x \in \Omega$.*

Theorem 9. *Assume that condition (18) is fulfilled. Let* $\{p_n\}_n \subset (N, \infty)$ *be a sequence of real numbers satisfying* $\lim_{n\to\infty} p_n = \infty$. *For each* $n > 1$ *denote by* $u_{p_n} \in W_0^{1,p_n}(\Omega)$ *a weak, nonnegative solution of problem* (19) *with* $p = p_n$. *Then the sequence* $\{u_{p_n}\}_n$ *converges uniformly in* Ω *to the distance function to the boundary of domain* Ω *given by* $\delta_H(x) := \inf_{y\in\partial\Omega} H^0(x-y)$, *for each* $x \in \Omega$, *where* $H^\circ(x) := \sup_{\xi\neq 0} \frac{\langle x,\xi\rangle}{H(\xi)}$, $\forall\, x, \xi \in \mathbb{R}^N$.

Chapter 4 presents some open problems related to the topic of this thesis which will guide our further research.

References

[1] M. Fărcăşeanu: An eigenvalue problem involving an anisotropic differential operator, *Complex Variables and Elliptic Equations*, **62** (2017), 297-306.

[2] M. Fărcăşeanu: On an eigenvalue problem involving the fractional (s,p)-Laplacian, *Fractional Calculus & Applied Analysis*, **21** (2018), 94-103.

[3] M. Fărcăşeanu, M. Mihăilescu: Continuity of the first eigenvalue for a family of degenerate eigenvalue problems, *Archiv der Mathematik*, **107** (2016), 659-667.

[4] M. Fărcăşeanu, M. Mihăilescu: On a family of torsional creep problems involving rapidly growing operators in divergence form, *Proceedings of the Royal Society of Edinburgh Section A: Mathematics*, in press.

[5] M. Fărcăşeanu, M. Mihăilescu & D. Stancu-Dumitru: A maximum principle for a class of first order differential operators, New Trends in Differential Equations, Control Theory and Optimisation, pp. 93-103, World Sci. Publ., Hackensack, NJ, (2016).

[6] M. Fărcăşeanu, M. Mihăilescu & D. Stancu-Dumitru: Perturbed fractional eigenvalue problems, *Discrete and Continuous Dynamical Systems-A*, **37** (2017), 6243-6255.

[7] M. Fărcăşeanu, M. Mihăilescu & D. Stancu-Dumitru: On the convergence of the sequence of solutions for a family of eigenvalue problems, *Mathematical Methods in the Applied Sciences*, **40** (2017), 6919-6926.

[8] M. Fărcăşeanu, M. Mihăilescu & D. Stancu-Dumitru: On a family of torsional creep problems in Finsler metrics, submitted.