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Resume

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Optimizations on differential systems of
cryptographic analysis for models which are
based on nonsupersingular elliptic curves.

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Thesis resume

The entire path of development of an differential cryptographic system reveals the beauty of pure mathematics and illustrates its applicability with real results in improving some parameters which have an important role in the final version of an cryptographic analysis system.

In the 1'st chapter we have the first step in the construction necessary for nonlinear asymmetric system, this being the study of the subspaces over which the nonsupersingular elliptic curves will be defined. In this regard, we start from the existing models and highlight the limitations, then we illustrate the particular subspaces which will be used in the developments from the next chapters. Let a a rational number which can be written as $a = q^m \frac{r}{s}$; $r \nmid q^k$, $s \nmid q^k$ and $a \neq 0$. We assign $\text{ord}_{q^k}(a) = m$ and obtain the following rule:

$$\text{ord}_{q^k}(a + b) \geq \min\{\text{ord}_{q^k}(a), \text{ord}_{q^k}(b)\},$$

as an equality, excepting the case $\text{ord}_{q^k}(a) = \text{ord}_{q^k}(b)$.

In the same manner for $a \in \mathbb{F}_{q^k}$, we assign $\text{ord}_{q^k}(a) = m$ if $a \in (q^k)^m \mathbb{Z}_{q^k} \setminus (q^k)^{m+1} \mathbb{Z}_{q^k}$. This rule also applies for the two definitions of ord_{q^k} sustain \mathbb{F}_{q^k} . In both cases we assigned $\text{ord}_{q^k}(0) = \infty$. We keep in mind that ord_{q^k} is an homeomorphism $\mathbb{F}_{q^k}^\times \rightarrow \mathbb{Z}$.

$E(\mathbb{F}_{q^k})$ has a compact topology and $E^0(\mathbb{F}_{q^k})$ is an open subset. Since $E(\mathbb{F}_{q^k})$ is a subsets union of $E^0(\mathbb{F}_{q^k})$, will result that there is only a finite set which fulfills the requirements.

Let $\mathbb{F}_{q^k} \times \mathbb{F}_{q^k} \times \mathbb{F}_{q^k}$ the topology result, $F_{q^k}^3 \setminus \{0, 0, 0\}$ the topology subset and $\mathbb{P}^2(\mathbb{F}_{q^k})$ the topology coefficient from $F_{q^k}^3 \setminus \{0, 0, 0\} \rightarrow \mathbb{P}^2(\mathbb{F}_{q^k})$. We have that $\mathbb{P}(\mathbb{F}_{q^k})$ is an association of images for subsets type $\mathbb{Z}_{q^k}^\times \times \mathbb{Z}_{q^k} \times \mathbb{Z}_{q^k}$, $\mathbb{Z}_{q^k} \times \mathbb{Z}_{q^k}^\times \times \mathbb{Z}_{q^k}$, $\mathbb{Z}_{q^k} \times \mathbb{Z}_{q^k} \times \mathbb{Z}_{q^k}^\times$, each one being compact and open then $\mathbb{P}^2(\mathbb{F}_{q^k})$ is compact. The subset $E(\mathbb{F}_{q^k})$ is closed because its the null set of an polynomial.

In relation to this topology on $\mathbb{P}(\mathbb{F}_{q^k})$, two points which are close will have the same reduction module q^k . So $E^0(\mathbb{F}_{q^k})$ is the space intersection $E(\mathbb{F}_{q^k})$ with an open subset from $\mathbb{P}^2(\mathbb{F}_{q^k})$.

It can be easily proven that reduction relation $E^0(\mathbb{F}_{q^k}) \rightarrow E(\mathbb{F}_{q^k})$ is surjective and is defined on the core $E1(\mathbb{F}_{q^k})$.

We assume that $E^n(\mathbb{F}_{q^k})$ is a subgroup from $E(\mathbb{F}_{q^k})$. If $\Omega = (x : y : 1)$ is in $E^1(\mathbb{F}_{q^k})$ then we will have $y \notin \mathbb{Z}_{q^k}$. Let $x = q^{-m} x_0$ and $y = (q^k)^{-m'} y_0$ with x_0 and y_0 from \mathbb{Z}_{q^k} .

Then

$$(q^k)^{-2m'} y_0^2 = (q^k)^{-3m} x_0^3 + a p^{-m} x_0 + b$$

From here the subspaces have been developed over which nonsupersingular elliptic curves are defined, which have as the base property the fact that the majority of points with cryptographic interest are proven that they are contained.

In the 2nd chapter, starting from the limitations of the existing systems, for particular cases needed in real implementation, we studied the possibility of extending the studies from the article [154], for the case of two users, we extended the study for the case of an group of users which use low performance devices. But my studies did not consist of reducing the complexity by optimizing the algorithms implementations, but through elaborating a mathematical model by taking into consideration the partitioning of a space over which are defined a set of particular elliptic curves, by this reducing the time needed to compute the parameters. Still maintaining the same Linear Equivalent Complexity of attack, achieved by way the space was partition over which the particular elliptic curves are defined.

The results were published in [60], [61], [62], [31].

In this regard, we developed a model of elliptic curves used in the particular system, described more in depth in the thesis.

Therefore, let E an elliptic curve defined as

$$Y^2 + \gamma_1XY + \gamma_3Y = X^3 + \gamma_2X^2 + \gamma_4X + \gamma_6$$

and $A_1 = (\omega_1, \eta_1)$, $A_2 = (\omega_2, \eta_2)$ two points on an elliptic curve defined in the described manner.

In this manner we can state that:

$$-A_1 = (\gamma_1, -\eta_1 - \gamma_1\omega_1 - \gamma_3)$$

where γ_6 is defined as nonlinear combination obtained from the start parameters used in encryption. From here we obtain

$$\lambda = \frac{\eta_2 - \eta_1}{\omega_2 - \omega_1}$$

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$$\gamma = \frac{\eta_1\omega_2 - \eta_2\omega_1}{\omega_2 - \omega_1}$$

where ω_1 and ω_2 satisfy the condition $\omega_1 \neq \omega_2$, which allowed to develop the next result:

$$\lambda = \frac{3\omega_1^2 + 2\alpha_2\omega_1 + \alpha_4 - \alpha_1\eta_1}{2\eta_1 + \alpha_1\omega_1 + \alpha_3}$$

și

$$\gamma = \frac{-\omega_1^3 + \alpha_4\omega_1 + 2\alpha_6 - \alpha_3\eta_1}{2\eta_1 + \alpha_1\omega_1 + \alpha_3}$$

An elliptic curve was defined over an subfraction of \mathbb{F}_q , in the following manner: $E(\mathbb{F}_{q^k})$. The curve can be easily deducted, it will contain m^2 points of order m , where m will divide $q^k - 1$, because having $E(m) \times E(m) \rightarrow \gamma_m$ where γ_m is a group of roots of order m of the unit, in K , will deduct the relation $div(g) = \sum_{D \in E(m)} (B'_1 + D) - (D)$ with $B' \in E(\bar{K})$, which fulfills the condition $[m]B' = B$. But, as stated in [11], we can have e_m as being:

$$e_m = \begin{cases} E(m) \times E(m) \rightarrow \gamma_m \\ (B_1, B_2) \rightarrow \frac{g(X+B_1)}{g(X)} \end{cases}$$

so the subspace determined by the fraction m will fulfill the expressed property, as stated in the formula and g will satisfy $g^2 - [t]g + [q] = [0]$.

Starting from the definition of the hierarchical communication access model [29] we will define a generating function of a public key set based on the conjugated information, where the space over which the elliptic curve is defined \mathbb{F}_{q^n} will have a multiplication factor K which will satisfy the relation $|K| \leq \lfloor q/2 \rfloor$. From here, corresponding to the level of the communication initiator (let him be A_i) from the user hierarchy, we will define a function like

$$\varphi(level, string) \rightarrow public\ Key$$

where string represents the initialization parameters of the generator, as described broadly in the thesis, and level represents the access level to the communication secure communication channel, from which A_i belongs.

In the sense of obtaining the interwoven encryption key, for a pair of participants, let them be (A_i, A_j) , they will create a session key if they are on the same level of security from the hierarchy, if they belong to different levels of security there will be a communication initiated by the owner of a higher security level, where these principles are broadly described in [154], [69], [71]. To describe them we will defined:

- $\Pi_{K_{A_i}}$ - the secret key of A_i
- $\Pi_{P_{A_i}}$ - the public key of A_i
- $\eta_{A_i}^d(\Pi_{K_{A_i}}, m)$ - message encryption m with the secret key of A_i
- $\eta_{A_i}^e(\Pi_{P_{A_i}}, m)$ - message encryption m with the public key of A_i
- $enc(s_K, m)$ - symmetric key for message encryption m together with s_K
- inf_{A_i} - pseudorandom value generated by A_i for every session
- $E(\mathbb{Z}_p)$ - the elliptic curve defined over the field \mathbb{Z}_p

- M - message space
- $hf(\cdot)$ - hash function $SHA - 1$
- $m_1|m_2$ - concatenation of messages m_1, m_2 when $m_1, m_2 \in M$

An user of the systems let him be A_i (with respect to the conditions expressed in the thesis) will have the following public parameters:

$$(\Pi_{P_{A_i}}, E(\mathbb{Z}_p), P, Q, n)$$

where $P, Q \in E(\mathbb{Z}_p)$ represents two points on the elliptic curve $E(\mathbb{Z}_p)$ and the division p , as stated in the thesis, will have the form q^k with respect to the presented conditions. Also, we will define the functions

- $\eta_{A_i}^d(\Pi_{K_{A_i}}, m)$ and
- $\eta_{A_i}^e(\Pi_{P_{A_i}}, m)$
- $hf(\cdot)$

as public.

For the user A_i , the following parameters are secret:

- $\Pi_{K_{A_i}}$
- $inf A_i$

Starting from the presented parameters, we can expose the protocol which establishes the session key between participants A_i and A_j .

- A_i
 1. We generate a random number $inf_{A_i} \in [1, n - 1]$
 2. We compute $A_i^1 = inf_{A_i}(P^{-1} + Q) = (x_1^{A_i}, y_1^{A_i})$. Let $x = x_1^{A_i} \bmod n$.
If $x = 0$ then we execute step 1
 3. We compute $A_i^2 = hf(P_{A_i}|A_i^1)$
 4. We compute $A_i^3 = \eta_{A_i}^d(\pi_{K_{A_i}}, A_i^2)$
 5. The first step of communication (from A_i to A_j)
 A_i sends to A_j ($A_i^1|A_i^2$)
- A_j
 1. We compute $A_j^1 = hf(P_{A_i}|A_i^1)$
 2. We compute $A_j^2 = \eta_{A_i}^e(\pi_{P_{A_i}}, A_i^2)$. If $A_j^1 \neq A_j^2$ terminates then the protocol ends with failure

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3. We generate a random number $\text{inf}_{A_j} \in [1, n - 1]$
 4. We compute $A_j^1 = \text{inf}_B(P^{-1} + Q) = (x_1^{A_j}, y_1^{A_j})$. If $x_1^{A_j} = 0$ then returns to step 3 from the steps executed by A_j
 5. We compute $A_j^2 = hf(P_{A_j} | A_j^1)$
 6. computes $A_j^3 = \eta_{A_j}^d(\pi_{K_{A_j}}, A_j^2)$
 7. $K_{A_j} = \text{inf}_{A_j} A_i^1 = (x_2^{A_j}, y_2^{A_j})$
 8. $x = x_2^{A_j} \bmod n$. If $x = 0$ then returns to step 3 from the steps executed by A_j
 9. The second step of communication (from A_j to A_i)
 A_j sends to A_i ($A_j^1 | A_j^3$)
- A_i
 - 6 We compute

$$s_1^{A_i} = hf(P_{A_j}, A_j^1)$$

$$s_2^{A_i} = \eta_{A_j}^e(\pi_{P_{A_i}}, A_j^3)$$
 - 7 If $s_1^{A_i} \neq s_2^{A_i}$ the execution of the protocol ends in failure.
 - 8 $K_{A_i} = \text{inf}_{A_i} A_j^1$

In order to ensure double authentication of the users involved in the process of secure communication, defined as A_i and A_j , we will define a third step to the described protocol.

Starting from the steps described in the optimized protocol, described in the thesis, there will be a third step which will ensure session key approval by A_i , by this it will be assured by the double authentication of the participants to the confidential communication channel.

In this regard, A_i will compute

$$hf((\text{inf}_{A_i}(P^{-1} + Q))) | \text{enc}(K_{A_i}, \text{inf}_{A_j}(P^{-1} + Q))$$

and will send the result to A_j .

At this point, when receiving the message, A_j will test the equality:

$$\begin{aligned} & hf((\text{inf}_{A_i}(P^{-1} + Q)) | \text{enc}(K_{A_i}, \text{inf}_{A_j}(P^{-1} + Q))) \\ &= hf((\text{inf}_{A_i}(P^{-1} + Q)) | \text{enc}(K_{A_j}, \text{inf}_{A_j}(P^{-1} + Q))). \end{aligned}$$

If the test of the equality will return success then the session key is confirmed by the participants. In implementing this system, we defined it as the conformation step. In regard to higher work speed it is used when the

participants are of different security levels, because in the case of an inequality at the third step this protocol will be restarted.

From a statistic point of view, the time needed to execute this step alongside with the complexity is of order $\Phi(1/4)$ from the required first two steps.

It was developed a personal version also for the extended algorithm, based on the mathematical model described above.

O version of the expressed protocol can be obtained by defining:

$$h'_{int}(h(k))$$

where $h'_{int} : M \rightarrow N$, h' representing the function which will generate a parameter $\eta \in \mathbb{N}$, $h'_{int}(h(k)) = \eta$, $\eta \in \mathbb{N}$, and η will respect the inequality $\eta \leq \sqrt{2} \cdot n$, where n represents the number of security levels. Let L_t , $0 \leq t \leq m$, the security levels. In this case, the key for every participant A_j^t will be created in two steps.

The first step of authentication of users which is fulfilled in the first part of the protocol and the second step - authenticating the key, is made at the additional step (third step).

This version starts form the idea of defining them as being different entities, the first are the participants to the process and the second the session keys used. Therefore it will be used a parametrization of $T.S.$, let it be defined as M_t^i called "master parametrization where t is the security level and i represents the participant index which initiates the communication process.

To highlight the functionality of the model it is necessary to prove the uniqueness of the parameters defined in the grade fraction q^k , over the elliptic curve used, more exactly the existence of the elliptical curve used.

In this sense, we will demonstrate the following theorem describing the parametrization used.

Theorem 1. Let Γ an nonsingular projection of an elliptic curve over fraction q^k , of type 1. In this case there is an elliptic curve, let $E(\mathbb{F}_{q^k})$ over q^k therefore Γ is an homogeneous space for $E(\mathbb{F}_{q^k})$ and $E(\mathbb{F}_{q^k})$ is unique defined by an isomorphism over q^k .

In the thesis there is the full demonstration of the above theorem which illustrates the properties of the used space.

In the 3rd chapter, starting from the idea of cryptographic systems used in session key generation we developed hierarchical models which treat a variety of cases of linear and nonlinear generators depending on the use case of each one. For the linear models, the applicability study can be reduced to the solution analysis of classic mathematical problems to which the Equivalent Linear Complexity is reduced. Regarding the resistance to cryptographic attacks, as a computer science model, they are stable. However, if you study the

computational effort with an mathematical analysis model based on solutions of atomic compounds constructed on bijections of the base model, reaching feasible models which are studied in real time.

In this regard, we developed the necessary model for stating and demonstrating the conjecture below, which facilitates the modeling illustration of an optimal system of optimal differential encryption.

Conjecture 1. Berlekamp-Massey for the case of compound dependencies.

For an equations system which describe the behavior a registry set with linear displacement dependencies of length λ , which will have as an output a sequence system

- (1) $\alpha_0, \alpha_1, \dots, \alpha_{N-1}$, for the linear case, where $\alpha_0 \neq 0, N \geq \alpha, \lambda'$ -the length of the generated string
- (2) $\alpha_0, \alpha_1, \dots, \alpha_{M-1}$, for the compound case, where $\alpha_0 \neq 0, M > N, \lambda''$ -the length of the generated string will satisfy the relations:
- (3) $\lambda' \geq N + 1 - \lambda, \lambda'' \geq M + 1 - \lambda$
- (4) $\lambda'' > \lambda'$

This is concretized by the following properties on the implementations of those parametrization

- Adding a parameter does not guarantee increasing the output string size / generator period, in other words, the type of comparison “o” from the Conjecture 1 is given by the type / grade of dependency between the initial parameters and the introduced parameter, let him be noted as: $DL(\lambda', \lambda'')$
- The ideal case, the one of dependence $DL = 0$ is transposed in the fact that the function “o” from the Conjecture will become the multiplication operation.

Starting from those, we developed an proprietary model of parametrization and construction of equations systems which define a set of shift registries with linear dependencies, named AGNS, which have an grater efficiency factor then the original model used in LFSR. E functional version will be:

$$\frac{C.L.E.}{Complex.Imp.}(AGNS) > \frac{C.L.E.}{Complex.Imp.}(LFSR)$$

where $C.L.E.$ is the equivalent linear complexity and $Complex.Imp.$ represents computation complexity of the implementation, for a generation system of

pseudorandom numbers. The results have been illustrated in the published article [31].

In this case we have:

$$\begin{aligned} b_k &\leq b_{k+1} \leq a_{k+1} \leq a_k \text{ și} \\ 0 &\leq a_{k+1} - b_{k+1} \leq (a_k - b_k)/2 \end{aligned}$$

From those expressed we can construct an proposition (results published in [31]) which illustrate that a iteration of type *AGM* constructs a sequence of elliptic curves based on a isomorphism of the initial elliptic curve.

Proposition 1. [31] Starting from choosing the two parameters a and b , so that $a, b \in 1 + 4\mathbb{Z}_q$ with the property $a/b \in 1 + 8\mathbb{Z}_q$ and an elliptic curve $E_{a,b}$ defined by the equation $y^2 = x(x - a^2)(x - b^2)$, let a' and b' , two parameters so that: $a' = (a + b)/2$, $b' = \sqrt{ab}$ and an elliptic curve $E_{a',b'}$, defined by equation $y^2 = x(x - a'^2)(x - b'^2)$. In this case, the elliptic curves $E_{a,b}$ and $E_{a',b'}$ are characterized by the equation:

$$\Phi : E_{a,b} \longrightarrow E_{a',b'} : (x, y) \longmapsto \left(\frac{(x + ab)^2}{4x}, y \frac{(x - ab)(x + ab)}{8x^2} \right)$$

and the greatest part of Φ is $\langle (0, 0) \rangle$. The operation Φ on the differential interval $\frac{dx}{y}$ will have the following form

$$\Phi^* \left(\frac{dx}{y} \right) = 2 \frac{dx}{y}.$$

In the 4th chapter we study the optimizations of the mathematical models used in group signing systems, therefore, starting from the concept of group signature presented by Chaum and van Heijst in the year 1991 ([18, 19, 22]), any member of the group can sign a message in behalf of the group so that anyone can verify the validity of the signature but no one can determine which member of the group sent the message ([40], [117], [168], [97], [98]).

From the developments made in the thesis, particular algorithms have resulted which are represented below.

All the models studied in the thesis and the described algorithms have been implemented in two research projects (UEFISCDI PCE and UEFISCDI PCCA) of which I am proud to be a member. Those results have been illustrated by testing and using the Digital Declaration system created, which is unique in Romania and the second system officially implemented at European level.

Algorithm 1 The key generation algorithm for a system derived from Schnorr

- 1: generating big prime numbers p and two points (P, Q) nonsingular on a nonsupersingular elliptic curve, described in the first chapter of the thesis
 - 2: g is the group generator
 - 3: the private key is selected x
 - 4: $y = g^x \pmod{p}$
 - 5: $\Phi = (p - 1)(q - 1)$
 - 6: the public key is (p, g, y, P)
 - 7: the private key is (p, g, x, Q)
-

Algorithm 2 The signing algorithm for a system derived from Schnorr

- 1: (p, g, x, P) is the private key
 - 2: is selected randomly k so that $0 < k < q$
 - 3: $r = P.x, g^k \pmod{p}$
 - 4: $e = H(m || r || P.y)$
 - 5: $s = (k - xe) \pmod{q}$
 - 6: the signature is (e, s)
-

Algorithm 3 The signature verification algorithm for a system derived from Schnorr

- 1: (p, g, y, Q) is the public key
 - 2: (e, s) is the signature
 - 3: $r_v = Q.y, g^s y^e$
 - 4: $e_v = H(m || r_v || Q.y)$
 - 5: if $(e_v = e)$ then
 - 6: (e, s) is valid
 - 7: end if
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